[illegible] - پاسخ تشریحی تکلیف ۱۵.

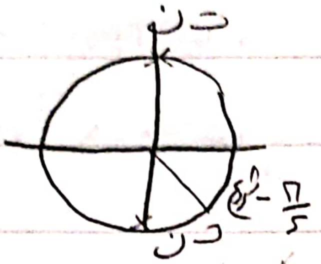
-۱

$y = \tan(-2x + \frac{\pi}{4})$ $\quad y = -\tan(2x - \frac{\pi}{4})$

$B = f(0) = -\tan(-\frac{\pi}{4}) = +1$ $\quad -2x + \frac{\pi}{4} = \frac{\pi}{2}$

$\frac{\pi}{4} - \frac{2\pi}{4} = 2x \to 2x = -\frac{\pi}{4}$

$x = -\frac{\pi}{8}$



$-2x + \frac{\pi}{4} = \frac{-\pi}{2} \to 2x = \frac{3\pi}{4}$

$S_{ABC} = \frac{\frac{\pi}{2} \times 1}{2} = \frac{\pi}{4}$ $\quad x = \frac{3\pi}{8}$

-۲

$x^2 - (\sin a)x - \frac{1}{4}\cos^2\alpha = 0$

$x^2 - (\sin a)x - \frac{1}{4}(1 - \sin^2\alpha) = 0$

$\xrightarrow{\frac{2}{3}} (\frac{2}{3})^2 - (\sin a)\frac{2}{3} - \frac{1}{4} + \frac{1}{4}\sin^2\alpha = 0$ $\quad (\frac{4}{9})$

$\cos^2\alpha = 1 - \sin^2\alpha$ $\quad \frac{1}{4}\sin^2\alpha - \frac{2}{3}\sin\alpha + \frac{7}{36} = 0$

$1 + \tan^2\alpha = \frac{1}{\cos^2\alpha} = \frac{1}{\frac{8}{9}} = \frac{9}{8}$ $\quad 9\sin^2\alpha - 24\sin\alpha + 7 = 0$

$\Delta = b^2 - 4ac = 576 - 252 = 324$

$\frac{16}{36} - \frac{9}{36} = \frac{7}{36}$

$\sin\alpha = \frac{24 \pm 18}{18}$ $\quad \frac{6}{18} = \frac{1}{3}$ $\quad \frac{42}{18} = \frac{7}{3}$ غ ق ق

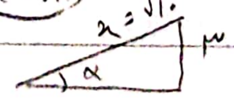
$\cos\alpha = \frac{2\sqrt{2}}{3}$

$\sin\alpha = \frac{1}{3}$ $\quad \to x^2 = 9 - 1 = 8$, $x = 2\sqrt{2}$

-۳

$\log(\sin a) - \log(-\cos\alpha) = \log 3$ $\quad x^2 = 9 + 1 = 10$, $x = \sqrt{10}$

$\log \frac{\sin a}{-\cos\alpha} = \log 3$



$\frac{\sin a}{-\cos a} = 3 \longrightarrow -\tan\alpha = 3 \longrightarrow \tan\alpha = -3$

$\underbrace{\sin a}_{-3\cos\alpha} - \cos\alpha = -4\cos\alpha = -4(\frac{1}{\sqrt{10}})$

$= \frac{-4}{\sqrt{10}}$.

۴- $\cos^4\theta + \sin^4\theta = ?$

$\sin^4\theta + \cos^2\theta = \frac{4}{5}$

$\sin^4\theta + 1 - \sin^2\theta = \frac{4}{5}$

$\sin^4\theta - \sin^2\theta = -\frac{1}{5}$

$\sin^2\theta(\underbrace{\sin^2\theta - 1}_{-\cos^2\theta}) = -\frac{1}{5}$

$\sin^2\theta\cos^2\theta = \frac{1}{5}$

$(\cos^2\theta + \sin^2\theta)^2 = 1$

$\sin^4\theta + \cos^4\theta + \cancel{2\sin^2\theta\cos^2\theta} =$

$\sin^4\theta + \cos^4\theta = \frac{3}{5}$

$-4\sin^4\theta - \sin^2\theta = -\frac{1}{5}$

$\boxed{\cos^4\theta + \sin^4\theta = \frac{3}{5}}$

۵- $\frac{3\sin x + \sin x\cos x}{2 - 2\cos^2 x} < 0 \; , \; \sin x\tan x - \frac{1}{\cos x} > 0$

$2(1-\cos^2 x) = 2\sin^2 x$

$\frac{\cancel{\sin x}\,(3 + \cos x)}{2\sin^2 x} < 0$

$\frac{3+\cos x}{2\sin x} < 0 \rightarrow \sin x < 0$

$\tan x\left(\sin x - \frac{1}{\sin x}\right) > 0$

$\tan x\left(\frac{\sin^2 x - 1}{\sin x}\right) > 0$

$\frac{\cancel{\sin x}}{\cos x}\left(\frac{\sin^2 x - 1}{\cancel{\sin x}}\right) > 0$

$-\frac{\cos^2 x}{\cos x} > 0 \rightarrow \cos x < 0$

ناحیه سوم

۶- $\frac{2}{\sin x} + \frac{3}{\cos x} = 0$

$\frac{2\cos x + 3\sin x}{\sin x\cos x} = 0 \longrightarrow 2\cos x + 3\sin x = 0$

$2\cos x = -3\sin x$

$-\frac{2}{3} = \tan x$

$-\frac{3}{2} = \cot x$

$\tan x + \cot x = -\frac{2}{3} - \frac{3}{2} = \frac{-4-9}{6} = -\frac{13}{6}$

$2\sin\alpha\cos\alpha$ $\qquad \sin 20$

$\sin^2 45 + \cos^2 45 + 2\sin 45\cos 45$ $\quad$ -۷

$$\frac{\sqrt{1+\sin 50}}{\sin 50 + \sin 10} =$$

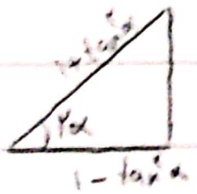
$$\underbrace{(1-2\sin^2 5)}_{\cos 10}\underbrace{(2\cos^2 10-1)}_{\cos 20}\underbrace{\left(\frac{1-\tan^2 20}{1+\tan^2 20}\right)}_{\cos 40} \quad -۸$$

$$\frac{2\sin 10}{2\sin 10}\times \underbrace{\underbrace{\cos 10\times\cos 20}_{\frac{1}{2}\sin 20}\times\cos 40}_{\frac{1}{2}\sin 40} \quad \frac{1}{2}\sin 80 \quad = \frac{\sin 80}{8\sin 10}$$

$$\frac{1}{8}\cot 10$$

$$\cos 2\alpha = 1-2\sin^2\alpha$$

$$\cos 2\alpha = 2\cos^2\alpha - 1$$



$$\rightarrow \cos 2\alpha = \frac{1-\tan^2\alpha}{1+\tan^2\alpha}$$

$$\sin\alpha + 3\cos\alpha = 2 \quad -۹$$

$$5\underbrace{\cos 2\alpha}_{2\cos^2\alpha-1} - 12\cos\alpha = 10\cos^2\alpha - 12\cos\alpha - 5$$

$$a\sin x + b\cos x = \frac{a}{|a|}\sqrt{a^2+b^2}\,\sin(x+\alpha) \qquad \tan\alpha = \frac{b}{a}$$

$$\sin\alpha + 3\cos\alpha = 1\sqrt{1^2+3^2}\,\sin(x+\alpha) \qquad \tan\alpha = 3$$

$$\sqrt{10}\,\sin(x+\alpha) = 2 \quad \tan\alpha = 3 \quad \longrightarrow \sin(x+\alpha) = \frac{2}{\sqrt{10}}$$

$\hat{A} = ۵$　　$\hat{B} = ۷۲^{\circ}$　　$\sin\hat{B}\,\sin\hat{C} = \cos^{۲}\frac{A}{۲}$　-۱۰

$۲\sin\hat{B}\,\sin\hat{C} = 1 + \cos A$

$$\cos^{۲}\frac{A}{۲} = \frac{1+\cos A}{۲}$$