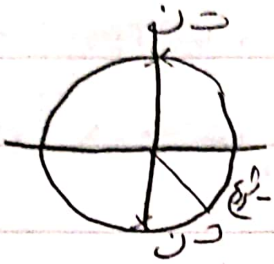
۱٤

ریاضیات – پاسخ تشریحی تکلیف ۱۵

۱– $y = \tan(-2x + \frac{\pi}{4})$ $y = -\tan(2x - \frac{\pi}{4})$

$B = f(0) = -\tan(-\frac{\pi}{4}) = +1$ $-2x + \frac{\pi}{4} = \frac{\pi}{2}$



$\frac{\pi}{4} - \frac{2\pi}{4} = 2x \to 2x = -\frac{\pi}{4}$

$x = -\frac{\pi}{8}$

$-\frac{\pi}{4}$ (۲)

$-2x + \frac{\pi}{4} = \frac{-\pi}{2} \to 2x = \frac{3\pi}{4}$

$S_{ABC} = \frac{\frac{\pi}{2} \times 1}{2} = \frac{\pi}{4}$ $x = \frac{3\pi}{8}$

۲– $x^2 - (\sin\alpha)x - \frac{1}{4}\cos^2\alpha = 0$

$x^2 - (\sin\alpha)x - \frac{1}{4}(1 - \sin^2\alpha) = 0$

$\frac{2}{3} \to (\frac{2}{3})^2 - (\sin\alpha)\frac{2}{3} - \frac{1}{4} + \frac{1}{4}\sin^2\alpha = 0$

$\cos^2\alpha = 1 - \sin^2\alpha$ (۲) $\frac{1}{4}\sin^2\alpha - \frac{2}{3}\sin\alpha + \frac{7}{36} = 0$

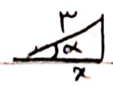
$1 + \tan^2\alpha = \frac{1}{\cos^2\alpha} = \frac{1}{\frac{8}{9}} = \frac{9}{8}$ $9\sin^2\alpha - 24\sin\alpha + 7 = 0$

$\Delta = b^2 - 4ac = 576 - 252 = 324$

$\frac{16}{36} - \frac{9}{36} = \frac{7}{36}$

$\cos\alpha = \frac{2\sqrt{2}}{3}$ $\sin\alpha = \frac{24 \pm 18}{18}$ $\frac{6}{18} = \frac{1}{3}$

$\sin\alpha = \frac{1}{3}$ $\frac{42}{18} = \frac{7}{3}$ غ ق ق



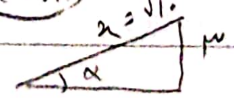
$x^2 = 9 - 1 = 8$ $x = 2\sqrt{2}$

۳– $\log(\sin\alpha) - \log(-\cos\alpha) = \log 3$

داخل لگاریتم $\sin\alpha > 0$ $\to$ $\log \sin$ $\log\frac{\sin\alpha}{-\cos\alpha} = \log 3$ $x^2 = 9 + 1 = 10$ $x = \sqrt{10}$

$-\cos\alpha > 0 \to \cos\alpha < 0$ $\to \log -\cos\alpha$

(۲) (۱٫۵)



$\frac{\sin\alpha}{-\cos\alpha} = 3 \to -\tan\alpha = 3 \to \tan\alpha = -3$

$\underbrace{\sin\alpha}_{-3\cos\alpha} - \cos\alpha = -4\cos\alpha = -4(-\frac{1}{\sqrt{10}})$

$\cos\alpha < 0$ $= +\frac{4}{\sqrt{10}}$

PAYCO

DATE :

CS CamScanner

**۴-**

$\cos^6\theta + \sin^6\theta = ?$

$\sin^4\theta + \cos^2\theta = \frac{4}{5}$

$\sin^4\theta + 1 - \sin^2\theta = \frac{4}{5}$

$\sin^4\theta - \sin^2\theta = -\frac{1}{5}$

$\sin^2\theta(\underbrace{\sin^2\theta - 1}_{-\cos^2\theta}) = -\frac{1}{5}$

$\sin^2\theta\cos^2\theta = \frac{1}{5}$

$(\cos^2\theta + \sin^2\theta)^3 = 1$

$\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta = 1$

$\sin^6\theta + \cos^6\theta = 1 - \frac{3}{5}$

$\boxed{\cos^6\theta + \sin^6\theta = \frac{2}{5}}$

(2)

**۵-**

$\frac{3\sin x + \sin x\cos x}{2 - 2\cos^2 x} < 0 \quad , \quad \sin x\tan x - \frac{1}{\cos x} > 0$

$2(1-\cos^2 x) = 2\sin^2 x$

$\frac{\sin x(3+\cos x)}{2\sin^2 x} < 0$

$\frac{3+\cos x}{2\sin x} < 0 \leftarrow \sin x < 0$

$\tan x\left(\sin x - \frac{1}{\sin x}\right) > 0$

$\tan x\left(\frac{\sin^2 x - 1}{\sin x}\right) > 0$

$\frac{\sin x}{\cos x}\left(\frac{\sin^2 x - 1}{\sin x}\right) > 0$

$\frac{-\cos^2 x}{\cos x} > 0 \rightarrow \cos x < 0$

(3)

ناحیه ۳

**۶-**

$\frac{2}{\sin x} + \frac{3}{\cos x} = 0$

$\frac{2\cos x + 3\sin x}{\sin x\cos x} = 0 \longrightarrow 2\cos x + 3\sin x = 0$

$2\cos x = -3\sin x$

$-\frac{2}{3} = \tan x$

$-\frac{3}{2} = \cot x$

$\tan x + \cot x = -\frac{2}{3} - \frac{3}{2} = \frac{-4-9}{6} = -\frac{13}{6}$

(4)

$2\sin\alpha\cos\alpha$

$\sin^2 25 + \cos^2 25 + 2\sin 25\cos 25$

$$\frac{\sqrt{1+\sin 50}}{\sin 50 + \sin 10} = \frac{\sqrt{1+\cos 40}}{\sin(30+20)+\sin(30-20)} \quad -7$$

$$\sqrt{2\cos^2 20} = \sqrt{2}\cos 20$$

$$\frac{\sqrt{2}\cos 20}{2\sin 30\cos 20} = \sqrt{2}$$

$$\underbrace{(1-2\sin^2 5)}_{\cos 10}\underbrace{(2\cos^2 10-1)}_{\cos 20}\underbrace{\left(\frac{1-\tan^2 20}{1+\tan^2 20}\right)}_{\cos 40} \quad -8$$

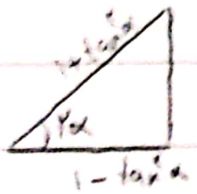
$$\frac{\overbrace{2\sin 10\times\cos 10}^{\sin 20}\times\cos 20\times\cos 40}{2\sin 10} = \frac{\sin 80}{8\sin 10}$$

$\frac{1}{2}\sin 40$, $\frac{1}{4}\sin 80$

$\frac{1}{8}\cot 10$

$\cos 2\alpha = 1-2\sin^2\alpha$

$\cos 2\alpha = 2\cos^2\alpha - 1$ (2)



$2\tan\alpha \rightarrow \cos 2\alpha = \dfrac{1-\tan^2\alpha}{1+\tan^2\alpha}$

$$\sin\alpha + 3\cos\alpha = 2 \quad -9$$

$$5\underbrace{\cos 2\alpha}_{2\cos^2\alpha-1} - 12\cos\alpha = 10\cos^2\alpha - 12\cos\alpha - 5$$

$a\sin x + b\cos x = \frac{a}{|a|}\sqrt{a^2+b^2}\,\sin(x+\alpha)$ $\quad \tan\alpha = \frac{b}{a}$

$\sin\alpha + 3\cos\alpha = 1\sqrt{1+3^2}\,\sin(x+\alpha)$ $\quad \tan\alpha = 3$

$\sqrt{10}\sin(x+\alpha) = 2 \longrightarrow \sin(x+\alpha) = \frac{2}{\sqrt{10}}$

15

$\sin\alpha = 2 - 3\cos\alpha$

$\sin^2\alpha = 4 + 9\cos^2\alpha - 12\cos\alpha$

$1-\cos^2\alpha = 9\cos^2\alpha - 12\cos\alpha + 4$

PAYCO

$10\cos^2\alpha - 12\cos\alpha + 3 = 0 \quad \left(\frac{1+\cos 2\alpha}{2}\right) - 12\cos\alpha + 3 = 0$

$5\cos 2\alpha - 12\cos\alpha = -8$

CamScanner

$\hat{A} = ?$ $\qquad \hat{B} = ۷۲^{\circ}$ $\qquad \sin\hat{B}\,\sin\hat{C} = \cos^{۲}\frac{A}{۲}$ -۱۰

$۲\sin\hat{B}\,\sin\hat{C} = ۱ + \cos A$

$\cos^{۲}\frac{A}{۲} = \frac{۱+\cos A}{۲}$

$۲\sin B\,\sin C = ۱ + \cos\hat{A}$ $\xrightarrow{A+B+C=\pi}$

$۲\sin B\,\sin C = ۱ + \cos(\pi - (B+C))$

$۲\sin B\,\sin C = ۱ - \cos(B+C)$

$\sin B\,\sin C + \cos B\,\cos C = ۱$

$\cos(B-C) = ۱ \longrightarrow B = C \longrightarrow$

$۱۸۰ - (۷۲+۷۲) = ۳۶$