تکلیف ۱۵:　　　　به نام خدا　　　　کیلی دستانی / دوازدهم دختر A

۱)

$y = \tan x \rightarrow T = \pi$ و $y = \tan(-2x + \frac{\pi}{4}) \rightarrow T = \frac{\pi}{-2} = -\frac{\pi}{2}$

$AC =$ اندازه یک دوره تناوب $\rightarrow AC = -\frac{\pi}{2}$

$B \Rightarrow x = 0 \rightarrow y = \tan(\frac{\pi}{4}) = 1 \rightarrow OB = 1$

$\rightarrow S = |AC \times OB \times \frac{1}{2}|$

$\rightarrow S = |-\frac{\pi}{2} \times 1 \times \frac{1}{2}| = +\frac{\pi}{4} \rightarrow 4$

۲)

$x^2 - (\sin\alpha)x - \frac{1}{4}\cos^2\alpha = 0 \xrightarrow{\cos^2\alpha = 1 - \sin^2\alpha}$

$x^2 - (\sin\alpha)x - \frac{1}{4}(1 - \sin^2\alpha) = 0 \rightarrow x^2 - (\sin\alpha)x + \frac{1}{4}\sin^2\alpha = \frac{1}{4}$

$\rightarrow (x - \frac{1}{2}(\sin\alpha))^2 = \frac{1}{4} \rightarrow x - \frac{1}{2}(\sin\alpha) = \frac{1}{2} \xrightarrow{x = \frac{2}{3}} \boxed{\sin\alpha = \frac{1}{3}}$ ق ق

$\rightarrow x - \frac{1}{2}(\sin\alpha) = -\frac{1}{2} \xrightarrow{x = \frac{2}{3}} \sin\alpha = \frac{7}{3}$ غ ق ق

$\boxed{* \; -1 \leq \sin\alpha \leq 1}$

$1 + \tan^2\alpha = \frac{1}{\cos^2\alpha} \rightarrow \cos^2\alpha = 1 - \sin^2\alpha = 1 - \frac{1}{9} = \frac{8}{9}$

$\boxed{\frac{1}{\cos^2\alpha} = \frac{9}{8}} \rightarrow 1.$

$\boxed{\sin\alpha > 0}$ $\quad -\cos\alpha > 0 \rightarrow \boxed{\cos\alpha < 0}$

۳) $\log^{(\sin\alpha)} - \log^{(-\cos\alpha)} = \log^{3} \rightarrow \log\left(\frac{\sin\alpha}{-\cos\alpha}\right) = \log^{3}$

$\rightarrow \frac{\sin\alpha}{\cos\alpha} = \boxed{\tan\alpha = -3} \rightarrow 1+\tan^2\alpha = \frac{1}{\cos^2\alpha} \rightarrow 1+9=10$

$\rightarrow \boxed{\cos\alpha = \frac{-1}{\sqrt{10}}}$ و $\sin^2\alpha = 1-\cos^2\alpha \rightarrow 1-\frac{1}{10} = \frac{9}{10}$

$\boxed{\sin\alpha = +\frac{3}{\sqrt{10}}} \Longrightarrow \sin\alpha - \cos\alpha = \frac{3}{\sqrt{10}} - \left(-\frac{1}{\sqrt{10}}\right) = \frac{4}{\sqrt{10}} \rightarrow$ ۲.

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۴) $\sin^4\theta + \cos^2\theta = \frac{4}{5} \xrightarrow{\cos^2\theta = 1-\sin^2\theta} \sin^4\theta - \sin^2\theta = -\frac{1}{5}$

$\rightarrow \sin^2\theta(\sin^2\theta - 1) = -\frac{1}{5} \rightarrow \boxed{\sin^2\theta\cos^2\theta = -\frac{1}{5}}$

$\cos^4\theta + \sin^2\theta = \cos^4\theta + 1 - \cos^2\theta = \cos^2\theta(\cos^2\theta - 1) + 1$

$= -\cos^2\theta\sin^2\theta + 1 = -\frac{1}{5} + 1 = \frac{4}{5} \rightarrow$ ۲.

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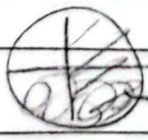
۵) $\frac{3\sin x + \sin x\cos x}{3 - 3\cos^2 x} = \frac{\sin x(\overbrace{3+\cos x}^{\text{+ مثبت}})}{3(1-\cos^2 x)} = \frac{\sin x(\overbrace{3+\cos x}^{\text{+ مثبت}})}{3\sin^2 x}$

$= \frac{\overbrace{3+\cos x}^{\text{+ مثبت}}}{3\sin x} < 0 \rightarrow 3\sin x < 0 \rightarrow \boxed{\sin x < 0}$

$\sin x\tan x - \frac{1}{\cos x} > 0 \rightarrow \frac{1}{\cos x}(\sin^2 x - 1) > 0 \rightarrow \frac{1}{\cos x}(\overbrace{-\cos^2 x}^{\text{منفی}}) > 0$

$\rightarrow \boxed{\cos x > 0} \Longrightarrow$  $\rightarrow$ ربع ۴ام $\rightarrow$ ۴.

$$\frac{2}{\sin x}+\frac{3}{\cos x}=0 \rightarrow \frac{2}{\sin x}=-\frac{3}{\cos x} \rightarrow 2\cos x=-3\sin x$$ ۶

$$\rightarrow \frac{\sin x}{\cos x}=\boxed{\frac{-2}{3}=\tan x} \;,\; \tan x=\frac{1}{\cot x} \rightarrow \boxed{\frac{-3}{2}=\cot x}$$

$$\tan x+\cot x=-\frac{2}{3}-\frac{3}{2}=\frac{-4-9}{6}=\frac{-13}{6} \rightarrow ۱$$

صورت $= \sqrt{1+\sin 50}=\sqrt{1+\cos 40}=\sqrt{1+\cos^2 20-\sin^2 20}$ ۷

$$=\sqrt{2\cos^2 20}=\cos 20\sqrt{2}$$

مخرج $= \sin 50+\sin 10=2\sin\left(\frac{50+10}{2}\right)\cos\left(\frac{50-10}{2}\right)$

$$=2\sin 30\cos 20=\cos 20$$

$$\Rightarrow \frac{\cos 20\sqrt{2}}{\cos 20}=\sqrt{2} \rightarrow ۱$$

$$\sin^2\alpha=\frac{1-\cos 2\alpha}{2} \rightarrow 2\sin^2\alpha=1-\cos 2\alpha \rightarrow 1-2\sin^2\alpha=\cos 2\alpha$$ ۸

$$\rightarrow 1-2\sin^2 5=\cos 10$$

$$\cos^2\alpha=\frac{1+\cos 2\alpha}{2} \rightarrow 2\cos^2\alpha=1+\cos 2\alpha \rightarrow 2\cos^2\alpha-1=\cos 2\alpha$$

$$\rightarrow 2\cos^2 10-1=\cos 20$$

$$\cos 2\alpha=\frac{1-\tan^2\alpha}{1+\tan^2\alpha} \rightarrow \frac{1-\tan^2 20}{1+\tan^2 20}=\cos 40$$

$$\Rightarrow (\cos 10\times\cos 20\times\cos 40)\times\frac{\sin 10}{\sin 10}=\frac{\frac{1}{8}\sin 80}{\sin 10}=\frac{1}{8}\frac{\cos 10}{\sin 10}$$

$$\xrightarrow{\cos 80=\sin 10} \frac{1}{8}\tan 80 \rightarrow ۲$$

$\sin\alpha + 3\cos\alpha = 2 \rightarrow \sin\alpha = 2 - 3\cos\alpha$ (9)

به توان ۲ $\rightarrow \sin^2\alpha = 4 + 9\cos^2\alpha - 12\cos\alpha \rightarrow 1 - \cos^2\alpha = 9\cos^2\alpha - 12\cos\alpha + 4$

$\rightarrow 10\cos^2\alpha - 12\cos\alpha = -3$

$5(\cos^2\alpha - \sin^2) - 12\cos\alpha = 5(2\cos^2\alpha - 1) - 12\cos\alpha =$

$\underbrace{10\cos^2\alpha - 12\cos\alpha}_{-3} - 5 = -8 \rightarrow 4$

$\sin B + \sin C = \cos^2\frac{A}{2} \rightarrow \sin B + \sin C = \frac{1+\cos A}{2}$ (10)

$\rightarrow 2\sin B\sin C = \cos A + 1$

$\rightarrow 2\left(\frac{1}{2}\left(\cos(B-C) - \cos(\cancelto{\pi-A}{B+C})\right)\right) = \cos A + 1$

$\rightarrow \cos(B-C) + \cos(A) = \cos A + 1 \rightarrow \cos(B-C) = 1$

$B - C = 0 \rightarrow B = C \rightarrow C = 72^\circ, B = 72^\circ$

$\Rightarrow 180 - (B+C) = 180 - 144 = \boxed{A = 36^\circ} \rightarrow 4$