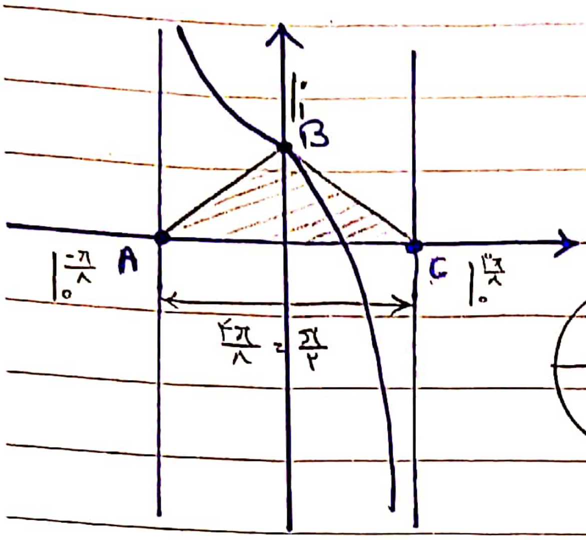
«بمیناهائت الحسنی» «دوازدهم دختر A تجربی»

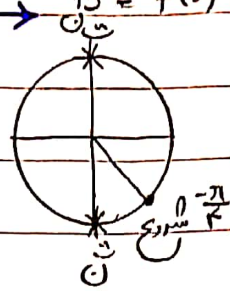
۱-



$y=\tan(-2x+\frac{\pi}{4})$

$y=-\tan(2x-\frac{\pi}{4})$

$B=f(0)=-\tan(\frac{-\pi}{4})=+1$



$-2x+\frac{\pi}{4}=\frac{\pi}{2}$

$\frac{\pi}{4}-\frac{2\pi}{4}=2x$

$-\frac{\pi}{4}=2x \Rightarrow x=-\frac{\pi}{8}$

$-2x+\frac{\pi}{4}=-\frac{\pi}{2} \Rightarrow 2x=\frac{3\pi}{4} \rightarrow x=\frac{3\pi}{8}$

$$S_{ABC}=\frac{\frac{\pi}{2}\times 1}{2}=\frac{\pi}{4}$$

۲-

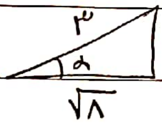
$x^2-(\sin\alpha)x-\frac{1}{4}\cos^2\alpha=0$ $(1-\sin^2\alpha)$ ریشه $=\frac{2}{3}$

$\frac{4}{9}-\frac{2}{3}\sin\alpha-\frac{1}{4}+\frac{1}{4}\sin^2\alpha=0$ $1+\tan^2\alpha=?!$

$\frac{1}{4}\sin^2\alpha-\frac{2}{3}\sin\alpha+\frac{7}{36}=0 \longrightarrow 9\sin^2\alpha-24\sin\alpha+7=0$

$\Delta'=144-63=81 \Rightarrow \sin\alpha=\frac{12\pm 9}{9}$ $\frac{21}{9}=\frac{7}{3}=2/1$ ✗ $\quad \frac{3}{9}=\frac{1}{3}$ ✓

$\sin\alpha=\frac{1}{3}$ $\quad 1+\tan^2x=\frac{1}{\cos^2x}$



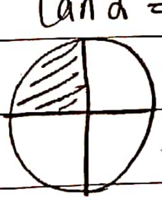
$\Rightarrow \cos\alpha=\frac{\sqrt{8}}{3} \Rightarrow \cos^2x=\frac{8}{9} \Rightarrow \frac{1}{\cos^2\alpha}=\frac{9}{8}$

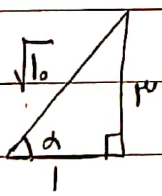
۳-

$\log(\sin\alpha)-\log(-\cos\alpha)=\log 3$ $\quad(\sin\alpha>0,\ \cos\alpha<0)$ $\Rightarrow \underbrace{\sin\alpha-\cos\alpha}_{-3\cos\alpha-\cos\alpha}=?!$

$\cancel{\log}\,\frac{-\sin\alpha}{\cos\alpha}=\cancel{\log}\,3 \Rightarrow \frac{\sin\alpha}{\cos\alpha}=-3$ $\quad -4\cos\alpha=?$

$\tan\alpha=-3$





$-4\cos\alpha=-4(-\frac{1}{\sqrt{10}})=\frac{4}{\sqrt{10}}=\frac{4\sqrt{10}}{10}$

$$\sin^4\theta + \underbrace{\cos^2\theta}_{1-\sin^2\theta} = \frac{4}{5} \Rightarrow \cos^4\theta + \sin^2\theta = ?!$$ ـ۴

$$(\sin^2\theta + \cos^2\theta)^2 = 1$$

$$\sin^4\theta + \cos^4\theta + \cancel{2\sin^2\theta\cos^2\theta}^{\frac{2}{5}} = 1$$

$$* \sin^4\theta - \sin^2\theta + 1 = \frac{4}{5}$$

$$\sin^2\theta(\underbrace{\sin^2\theta - 1}_{-\cos^2\theta}) = \frac{-1}{5}$$

$$\sin^2\theta\cos^2\theta = \frac{1}{5}$$

$$\ominus \begin{cases} \sin^4\theta + \cos^4\theta = \frac{3}{5} \\ * \sin^4\theta - \sin^2\theta = \frac{-1}{5} \end{cases}$$

$$\boxed{\cos^4\theta + \sin^2\theta = \frac{4}{5}}$$

$(\sin\alpha, \cos\alpha \neq 0)$ ـ۵

$$\sin\alpha\tan\alpha - \frac{1}{\cos\alpha} > 0$$

$$\frac{3\sin\alpha + \sin\alpha\cos\alpha}{\underbrace{2 - 2\cos^2\alpha}_{+}} < 0$$

$0 \leq \cos^2\alpha \leq 1$

$0 \leq 2\cos^2\alpha \leq 2$

$-2 \leq -2\cos^2\alpha \leq 0$

$0 \leq 2 - 2\cos^2\alpha \leq 2$

$$\Rightarrow 3\sin\alpha + \sin\alpha\cos\alpha < 0$$

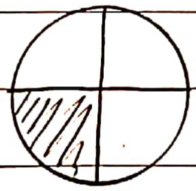
$$\sin\alpha\underbrace{(3 + \cos\alpha)}_{+} < 0 \Rightarrow \boxed{\sin\alpha < 0}$$

$-1 \leq \cos\alpha \leq 1$

$2 \leq 3 + \cos\alpha \leq 4$

$$\tan\alpha\left(\sin\alpha - \frac{1}{\sin\alpha}\right) > 0 \Rightarrow \tan\alpha\left(\frac{\sin^2\alpha - 1}{\sin\alpha}\right) > 0$$

$$\frac{\cancel{\sin\alpha}}{\cos\alpha}\left(\frac{\sin^2\alpha - 1}{\cancel{\sin\alpha}}\right) > 0 \Rightarrow \frac{-\cos^2\alpha}{\cos\alpha} > 0 \Rightarrow \boxed{\cos\alpha < 0}$$



$\Rightarrow$ * α در ربع سوم دایره مثلثاتی است

$\tan\alpha + \cot\alpha = ?!$ ـ۶

$$\frac{2}{\sin\alpha} + \frac{3}{\cos\alpha} = 0$$

$$\frac{2\cos\alpha + 3\sin\alpha}{\sin\alpha\cos\alpha} = 0 \Rightarrow 2\cos\alpha + 3\sin\alpha = 0 \Rightarrow 2\cos\alpha = -3\sin\alpha$$

$$\begin{cases} \tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{-2}{3} \\ \cot\alpha = \frac{\cos\alpha}{\sin\alpha} = \frac{-3}{2} \end{cases} \Rightarrow \frac{-2}{3} - \frac{3}{2} = \frac{-4-9}{6} = \frac{-13}{6}$$

$* \cos 20°$

$$\frac{\sqrt{1+\cancel{\sin 50°}\ \cos 40°}}{\sin 50° + \sin 10°} \rightsquigarrow \sqrt{1+\cos 40°} = \sqrt{2\cos^2 20°} = \sqrt{2}\cos 20°$$ -۷

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \Rightarrow \sin 50° + \sin 10° =$$

$$2\sin 30°\cos 20° = 2\times\frac{1}{2}\times\cos 20° = \cos 20°$$

$$\Rightarrow \frac{\sqrt{2}\,\cancel{\cos 20°}}{\cancel{\cos 20°}} = \boxed{\sqrt{2}}$$

$$\underbrace{(1-2\sin^2 5°)}_{\cos 10°}\underbrace{(2\cos^2 10° - 1)}_{\cos 20°}\underbrace{\left(\frac{1-\tan^2 20°}{1+\tan^2 20°}\right)}_{\cos 40°}$$ -۸

$$\frac{\underbrace{\underbrace{\underbrace{\sin 10°\cos 10°}_{\frac{1}{2}\sin 20°}\cos 20°}_{\frac{1}{2}\sin 40°}\cos 40°}_{\frac{1}{2}\sin 80°}}{\sin 10°} \Rightarrow \frac{\frac{1}{8}\sin 80°}{\sin 10°} = \frac{1}{8}\,\frac{\cos 10°}{\sin 10°} = \frac{1}{8}\cot 10°$$

$$\sin\alpha + 3\cos\alpha = 2 \Rightarrow 5\cos^2\alpha - 12\cos\alpha = ?!$$ -۹

$$\sin\alpha = 2 - 3\cos\alpha$$

$$\sin^2\alpha + \cos^2\alpha = 4 + 9\cos^2\alpha - 12\cos\alpha + \cos^2\alpha$$

$$1 = 4 + 10\cos^2\alpha - 12\cos\alpha$$

$$10\cos^2\alpha - 12\cos\alpha = -3$$

$$5(2\cos^2\alpha - 1) - 12\cos\alpha =$$

$$10\cos^2\alpha - 5 - 12\cos\alpha =$$

$$10\cos^2\alpha - 12\cos\alpha - 5 = ?$$

$$\Rightarrow 10\cos^2\alpha - 12\cos\alpha - 5 = -8$$

$$\sin\hat{B}\sin\hat{C} = \cos^2\frac{\hat{A}}{2} \qquad \hat{B} = 72° \Rightarrow \hat{A} = ?$$ -۱۰

$$\hat{A} + \hat{B} + \hat{C} = 180° \rightsquigarrow \hat{B} = 72° \Rightarrow \hat{A} + \hat{C} = 180 - 72 = 108° \Rightarrow \hat{C} = 108° - \hat{A}$$

$$\underbrace{\sin 72° \times \sin(108 - \hat{A})} = \cos^2\frac{\hat{A}}{2}$$

$$\frac{1}{2}\left[\cos(A-36) - \underbrace{\cos(180° - A)}_{-\cos A}\right] = \cos^2\frac{\hat{A}}{2}$$

$$\frac{1}{2}\left[\cos(A - 36°) + \cos A\right] = \cos^2\frac{A}{2} \rightarrow \frac{1+\cos A}{2}$$

$$\cos(A-36) + \cancel{\cos A} = 1 + \cancel{\cos A} \Rightarrow \cos(A-36) = 1$$

$$A - 36° = 0 \Rightarrow \boxed{\hat{A} = 36°}$$