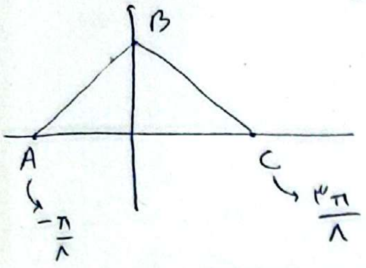
۱)

$y=\tan(-2x+\frac{\pi}{4}) \to B: x=0 \to \tan\frac{\pi}{4}=1 \to B(0,1)$

$\to A, C: \tan(-2x+\frac{\pi}{4})=$ تن $\to \tan(-2x+\frac{\pi}{4})=\tan(\frac{\pi}{2}) \to -2x=\frac{\pi}{2}-\frac{\pi}{4} \to x=-\frac{\pi}{8}$ A

$\to \tan(-2x+\frac{\pi}{4})=\tan(-\frac{\pi}{2}) \to -2x=-\frac{\pi}{2}-\frac{\pi}{4} \to x=\frac{3\pi}{8}$ C



$\to \frac{\frac{3\pi}{8}+\frac{\pi}{8}}{2}=\frac{\pi}{4}$

$C-A=T=\frac{\pi}{2}$

۲)

$x^2-(\sin\alpha)x-\frac{1}{4}\cos^2\alpha=0 \xrightarrow{x=\frac{2}{3}} \frac{4}{9}-\frac{2}{3}\sin\alpha-\frac{1}{4}(1-\sin^2\alpha)=0$

$\to 16-24\sin\alpha-9(1-\sin^2\alpha)=9\sin^2\alpha-24\sin\alpha+7=0 \to$

$\sin\alpha=\frac{24\pm\cancel{\sqrt{324}}^{\,18}}{18} \to \frac{42}{18}$ ✗

$\to \frac{6}{18}=\frac{1}{3}$ ✓ $\to \cos\alpha=\sqrt{1-\sin^2\alpha}=\sqrt{1-\frac{1}{9}}=\frac{2\sqrt{2}}{3} \to$

$1+\tan^2\alpha=\frac{1}{\cos^2\alpha} \to \frac{1}{\cos^2\alpha}=\frac{1}{\frac{8}{9}}=\boxed{\frac{9}{8}}$

۳)

$\log(\sin\alpha)-\log(-\cos\alpha)=\log 3 \to \log\frac{\sin\alpha}{-\cos\alpha}=\log 3 \to \sin\alpha=-3\cos\alpha$

$\sin^2\alpha+\cos^2\alpha=1 \to 9\cos^2+\cos^2\alpha=1 \to \cos^2\alpha=\frac{1}{10} \to \cos\alpha=\frac{-\sqrt{10}}{10} \to \sin\alpha=\frac{3\sqrt{10}}{10} \to$

$\sin\alpha-\cos\alpha=\frac{3\sqrt{10}}{10}+\frac{\sqrt{10}}{10}=\frac{4\sqrt{10}}{10}=\frac{2\sqrt{10}}{5}$

۴)

$\sin^4\theta+\cos^2\theta=\frac{4}{5} \to \sin^4\theta+1-\sin^2\theta=\frac{4}{5} \to \sin^2\theta(\sin^2\theta-1)=-\frac{1}{5} \to$

$\sin^2\theta\cos^2\theta=\frac{1}{5}$

$\cos^4\theta+\sin^2\theta=\cos^4\theta+1-\cos^2\theta=\cos^2\theta(\cos^2\theta-1)+1=-\cancel{\sin^2\theta\cos^2\theta}^{\,-\frac{1}{5}}+1=\boxed{\frac{4}{5}}$

۵)

$\frac{3\sin x+\sin x\cos x}{2-2\cos^2x}<0 \to \frac{\sin x(3+\cos x)}{2\underbrace{\cancel{(1-\cos^2x)}}_{\sin^2x}}=\frac{\overbrace{3+\cos x}^{\text{همواره }+}}{\underbrace{2\sin x}_{\sin\neq 0}}<0 \to 2\sin x<0 \to \sin x<0$

$\sin x\tan x-\frac{1}{\cos x}>0 \to \frac{1}{\cos x}\underbrace{\cancel{(\sin^2x-1)}}_{-\cos^2x}>0 \xrightarrow[\cos x\neq 0]{} -\cos x>0 \to \cos x<0$

ناحیه سوم

۶)

$$\frac{2}{\sin x}+\frac{3}{\cos x}=0 \longrightarrow \frac{2\cos x+3\sin x}{\sin x\cos x}=0 \longrightarrow 2\cos x+3\sin x=0 \longrightarrow \cos x=-\frac{3}{2}\sin x$$

$$\sin^2 x+\cos^2 x=1 \longrightarrow \sin^2 x+\frac{9}{4}\sin^2 x=1 \longrightarrow \sin^2 x=\frac{4}{13} \longrightarrow \sin x=\pm\frac{2\sqrt{13}}{13}$$

$$\longrightarrow \cos x=\pm\frac{3\sqrt{13}}{13}$$

مختلف العلامت هستند

$$\downarrow$$

$$\sin x\cos x<0$$

$$\tan x+\cot x=\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}=\frac{1}{\sin x\cos x}=\frac{1}{-\frac{2\sqrt{13}}{13}\times\frac{3\sqrt{13}}{13}}=\boxed{\frac{-13}{6}}$$

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۷)

$$1+\sin 50=1+2\sin 25\cos 25=(\sin 25+\cos 25)^2 \longrightarrow \sqrt{1+\sin 50}=\sin 25+\cos 25$$

$$\sin\alpha+\cos\alpha=\sqrt{2}\,\sin\left(\alpha+\frac{\pi}{4}\right) \longrightarrow \sin 25+\cos 25=\sqrt{2}\,\sin 70=\sqrt{2}\cos 20$$

$$\sin 50+\sin 10=\sin(20+30)+\sin(30-20)=\sin 20\cos 30+\sin 30\cos 20+\sin 30\cos 20-\cos 30\sin 20$$

$$2\cancel{\sin 30}^{\frac{1}{2}}\cos 20=\cos 20 \longrightarrow \frac{\sqrt{1+\sin 50}}{\sin 50+\sin 10}=\frac{\sqrt{2}\cos 20}{\cos 20}=\sqrt{2}$$

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۸)

$$\sin^2\alpha=\frac{1-\cos 2\alpha}{2} \longrightarrow \cos 2\alpha=1-2\sin^2\alpha \longrightarrow 1-2\sin^2 5=\cos 10$$

$$\cos^2\alpha=\frac{1+\cos 2\alpha}{2} \longrightarrow \cos 2\alpha=2\cos^2\alpha-1 \longrightarrow 2\cos^2 10-1=\cos 20$$

$$\cos 2\alpha=\frac{1-\tan^2\alpha}{1+\tan^2\alpha} \longrightarrow \frac{1-\tan^2 20}{1+\tan^2 20}=\cos 40 \longrightarrow$$

$$(1-2\sin^2 5)(2\cos 10-1)\left(\frac{1-\tan^2 20}{1+\tan^2 20}\right)=\frac{\sin 10}{\sin 10}\times\cos 10\times\cos 20\times\cos 40=\frac{1}{8}\,\frac{\sin 80}{\sin 10} \longrightarrow$$

$$\frac{1}{8}\times\frac{\cos 10}{\sin 10}=\boxed{\frac{1}{8}\cot 10}$$

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۹)

$$\cos 2\alpha=2\cos^2\alpha-1 \longrightarrow 5\cos 2\alpha-12\cos\alpha=10\cos^2\alpha-12\cos\alpha-5=?$$

$$\sin\alpha+3\cos\alpha=2 \longrightarrow \sin\alpha=2-3\cos\alpha \longrightarrow \sin^2\alpha+\cos^2\alpha=1$$

$$(2-3\cos\alpha)^2+\cos^2\alpha=9\cos^2\alpha+4-12\cos\alpha+\cos^2\alpha=10\cos^2\alpha-12\cos\alpha+4=1 \longrightarrow$$

$$10\cos^2\alpha-12\cos\alpha=-3 \longrightarrow 10\cos^2\alpha-12\cos\alpha-5=-3-5=\boxed{-8}$$

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۱۰)

$$\cos^2\frac{A}{2}=\frac{1+\cos A}{2} \qquad \cos A=\cos(\pi-(B+C))=-\cos(B+C) \longrightarrow$$

$$1-\cos(B+C)=2\sin B\sin C \longrightarrow 1-(\cos B\cos C-\sin B\sin C)=2\sin B\sin C \longrightarrow$$

$$1-\cos B\cos C+\sin B\sin C=2\sin B\sin C \longrightarrow$$

$$1-(\cos B\cos C+\sin B\sin C)=0 \longrightarrow 1-\cos(B-C)=0 \longrightarrow \cos(B-C)=1 \longrightarrow B-C=0 \rightarrow B=C=72$$

$$\hat{A}=180-(\hat{B}+\hat{C})=180-144 \longrightarrow \boxed{A=36^\circ}$$