نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۱۵ .. کلاس ..... A

۱

$y=\tan(-2x+\frac{\pi}{4}) \xrightarrow{x=0} \tan(\frac{\pi}{4})=1 \Rightarrow B(0,1)$

$\tan(-2x+\frac{\pi}{4})=\frac{\sin(-2x+\frac{\pi}{4})}{\cos(-2x+\frac{\pi}{4})} \rightarrow \cos(-2x+\frac{\pi}{4})=0 \Rightarrow \begin{cases} -2x+\frac{\pi}{4}=\frac{\pi}{2} \Rightarrow x=-\frac{\pi}{8} \rightarrow A \\ -2x+\frac{\pi}{4}=-\frac{\pi}{2} \Rightarrow x=\frac{3\pi}{8} \rightarrow C \end{cases}$

$\overline{AC}=\frac{\pi}{2} \Rightarrow S_{ABC}=\frac{\pi}{2}\times 1\times\frac{1}{2}=\boxed{\frac{\pi}{4}}$ ✓

۲

$x^2-(\sin\alpha)x-\frac{1}{4}\cos^2\alpha=0 \xrightarrow{x=\frac{1}{2}} \frac{1}{4}-\frac{1}{2}\sin\alpha-\frac{1}{4}\cos^2\alpha=0 \xrightarrow[\text{در ۴ ضرب کنید}]{\cos^2\alpha=1-\sin^2\alpha} 9\sin^2\alpha-4\sin\alpha+1=0$

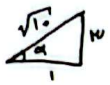
$\Rightarrow \begin{cases} \sin\alpha=\frac{1}{3} \rightarrow \text{قبول} \\ \sin\alpha=\frac{1}{3} \rightarrow \end{cases}$  $\Rightarrow \cos\alpha=\frac{\sqrt{8}}{3}$ / $\tan^2\alpha+1=\frac{1}{\cos^2\alpha}=\frac{1}{\frac{8}{9}}=\boxed{\frac{9}{8}}$

✓

۳

$\log_{\sin\alpha}^{\sin\alpha} - \log_{-\cos\alpha}^{-\cos\alpha} = \log^3 \Rightarrow \log^{-\tan\alpha} = \log^3 \Rightarrow \tan\alpha=-3$

$\sin\alpha>0$, $-\cos\alpha>0 \Rightarrow \cos\alpha<0$



$\Rightarrow \sin\alpha=\frac{3}{\sqrt{10}}=\frac{3\sqrt{10}}{10}$, $\cos\alpha=-\frac{\sqrt{10}}{10}$ / $\sin\alpha-\cos\alpha=\frac{3\sqrt{10}}{10}+\frac{\sqrt{10}}{10}=\frac{4\sqrt{10}}{10}=\boxed{\frac{2\sqrt{10}}{5}}$

✓

۴

$\sin^4\theta+\cos^2\theta-\frac{1}{5}=0 \xrightarrow{\cos^2\theta=1-\sin^2\theta} \sin^4\theta-\sin^2\theta+\frac{1}{5}=0 \Rightarrow \begin{cases} \sin^2\theta=\frac{5+\sqrt{5}}{10} \Rightarrow \cos^2\theta=\frac{5-\sqrt{5}}{10} \\ \sin^2\theta=\frac{5-\sqrt{5}}{10} \Rightarrow \cos^2\theta=\frac{5+\sqrt{5}}{10} \end{cases}$

$\begin{cases} \cos^4\theta+\sin^2\theta=\frac{25+5-10\sqrt{5}}{100}+\frac{5+\sqrt{5}}{10}=1/1 \\ \cos^4\theta+\sin^2\theta=\frac{25+5+10\sqrt{5}}{100}+\frac{5-\sqrt{5}}{10}=1/1 \end{cases}$

$\Rightarrow \cos^4\theta+\sin^2\theta=\boxed{1/1}$

✓

۵

$\frac{3\sin\alpha+\sin\alpha\cos\alpha}{2-2\cos^2\alpha}<0 \rightarrow \frac{\sin\alpha(\underbrace{3+\cos\alpha}_{\text{همواره }(+)})}{2(1-\cos^2\alpha)}<0 \Rightarrow \sin\alpha<0 \quad (1)$

$\sin^2\alpha$ مخرج همواره مثبت

(1) و (2) $\Rightarrow$ ناحیه سوم

$\sin\alpha\tan\alpha-\frac{1}{\cos\alpha}>0 \Rightarrow -\cos\alpha>0 \Rightarrow \cos\alpha<0 \quad (2)$

✓

$\frac{2}{\sin x} = \frac{-3}{\cos x} \Rightarrow 2\cos x = -3\sin x \Rightarrow \cos x = -\frac{3}{2}\sin x \quad (*)$

$\sin^2 + \cos^2 x = 1 \xrightarrow{(*)} \frac{13}{4}\sin^2 x = 1 \Rightarrow \sin^2 x = \frac{4}{13} \,,\ \cos^2 x = \frac{9}{13} \Rightarrow \sin x = \frac{2}{\sqrt{13}}$ ۶

$, \cos x = \frac{-3}{\sqrt{13}}$

$\tan x + \cot x = \frac{1}{\sin x \cos x} = \frac{1}{\frac{2}{\sqrt{13}} \times \frac{-3}{\sqrt{13}}} = \frac{13}{-6}$ ۲ ✓

$\frac{\sqrt{1+\sin 50^\circ}}{\sin 50^\circ + \sin 10^\circ} = \frac{\sqrt{(\sin 25^\circ + \cos 25^\circ)^2}}{\sin 50^\circ + \sin 10^\circ} = \frac{|\sin 25^\circ + \cos 25^\circ|}{\sin 50^\circ + \sin 10^\circ} = \frac{\sqrt{2}\cos 20^\circ}{\cos 20^\circ} = \sqrt{2}$

$-\sin 50^\circ = \sin(30^\circ + 20^\circ) = \sin 30^\circ \cos 20^\circ + \cos 30^\circ \sin 20^\circ$

$-\sin 10^\circ = \sin(30^\circ - 20^\circ) = \sin 30^\circ \cos 20^\circ - \cos 30^\circ \sin 20^\circ$ ۷

$\Rightarrow \sin 50^\circ + \sin 10^\circ = 2\sin 30^\circ \cos 20^\circ = \cos 20^\circ \quad (\sin 30^\circ = \frac{1}{2})$

$-\sin 25^\circ + \cos 25^\circ = \sqrt{2}\sin(25^\circ + 45^\circ) = \sqrt{2}\sin 70^\circ = \sqrt{2}\cos 20^\circ$ ۲ ✓

$(1 - 2\sin^2 5^\circ)(2\cos^2 10^\circ - 1)\left(\frac{1-\tan^2 20^\circ}{1+\tan^2 20^\circ}\right) \times \frac{\sin 10^\circ}{\sin 10^\circ} = \frac{1}{2}\sin 20^\circ \cos 20^\circ \cos 40^\circ = \frac{1}{8}\sin 80^\circ$

$(1 - 2\sin^2 5^\circ) \to \cos 10^\circ ,\ (2\cos^2 10^\circ - 1) \to \cos 20^\circ ,\ \frac{1-\tan^2 20^\circ}{1+\tan^2 20^\circ} \to \cos 40^\circ$

$\frac{1}{2}\sin 20^\circ \cos 20^\circ \to \frac{1}{4}\sin 40^\circ \to \frac{1}{8}\sin 80^\circ$ ۸

$\Rightarrow \frac{\sin 80^\circ}{8\sin 10^\circ} = \frac{\cos 10^\circ}{8\sin 10^\circ} = \boxed{\frac{1}{8}\cot 10^\circ}$ ۲ ✓

$\sin\alpha + 3\cos\alpha = 2 \Rightarrow \sin\alpha = 2 - 3\cos\alpha$

$\cos^2\alpha + \sin^2\alpha = 1 \Rightarrow \cos^2\alpha + 9\cos^2\alpha - 12\cos\alpha + 4 = 1 \Rightarrow 10\cos^2\alpha - 12\cos\alpha + 3 = 0$

$\Rightarrow \cos\alpha = \frac{12 \pm 2\sqrt{6}}{20} = \frac{3 \pm \sqrt{6}}{5} \qquad 10\cos^2\alpha - 12\cos\alpha = -3$ ۹

$5\cos^2\alpha - 12\cos\alpha =$

$10\cos^2\alpha - 12\cos\alpha - 5 = -8$

۲ ✓

$\cos^2\frac{A}{2} = \frac{1+\cos A}{2} \qquad 2\sin B \sin C = 1 + \cos A \quad \underline{A+B+C = 180}$

$2\sin B \sin C = 1 - \cos(180 - (B+C)) \to \cos(B+C)$ ۱۰

$\sin B \sin C + \cos B \cos C = 1 \to \cos(B - C) = 1$

$\to B = C \to 180 - (72 + 72) = 36^\circ$