نام و نام خانوادگی .... پاسخنامه تشریحی تکلیف شماره ..۱۵.. کلاس ....

۱۹/۵

۱

$\tan(\frac{\pi}{4}) = 1 \rightarrow B(0, 1)$

$\tan(-2x + \frac{\pi}{4}) = 1 \rightarrow -2x + \frac{\pi}{4} = -\frac{\pi}{2} \rightarrow x = \frac{3\pi}{8}$

$\quad\quad \rightarrow -2x + \frac{\pi}{4} = \frac{\pi}{2} \rightarrow x = -\frac{\pi}{8}$

$\Big\}$ AC طول $= \frac{\pi}{4}$

$S_{ABC} = \frac{1}{2} AC \times BO = \frac{1}{2} \times \frac{\pi}{4} \times 1 = \frac{\pi}{8}$

۲ ✓

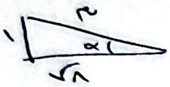
۲

$\frac{4}{9} - \frac{2}{3}(\sin\alpha) - \frac{1}{4}(1 - \sin^2\alpha) = 0 \rightarrow \frac{\sin^2\alpha}{4} - \frac{2\sin\alpha}{3} + \frac{7}{36} = 0 \rightarrow$

$9\sin^2\alpha - 24\sin\alpha + 7 = 0 \rightarrow \sin^2\alpha - 24\sin\alpha + 63 = 0 \rightarrow (\sin\alpha - 21)(\sin\alpha - 3) = 0 \rightarrow \sin\alpha = \cancel{\frac{21}{9}} = \frac{3}{9}$

غ ق ق

$\sin\alpha = \frac{1}{3}$

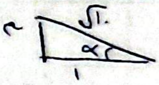


$\tan\alpha = \frac{1}{\sqrt{8}}, \quad \tan^2\alpha + 1 = \frac{1}{8} + 1 = \frac{9}{8}$

۲ ✓

۳

$\log \sin\alpha - \log(-\cos\alpha) = \log 3 \rightarrow \log \frac{\sin\alpha}{-\cos\alpha} = \log 3 \rightarrow -\tan\alpha = 3$



$\sin\alpha = \frac{1}{\sqrt{10}}, \quad \cos\alpha = \frac{-3}{\sqrt{10}}$

با توجه به دامنه لگاریتم فقط $\cos\alpha$ منفی است :

$\sin\alpha - \cos\alpha = \frac{1}{\sqrt{10}} + \frac{3}{\sqrt{10}} = \frac{4}{\sqrt{10}}$

۲ ✓

۴

$\cos^4\alpha - \sin^4\alpha = \cos^2\alpha - \sin^2\alpha$ ①

$\sin^4\alpha - \cos^4\alpha = \sin^2\alpha - \cos^2\alpha$ ②

$\sin^4\alpha + \cos^4\alpha = \frac{4}{5} \xrightarrow{\text{۲.ط}} \sin^2\alpha - \cancel{\cos^2\alpha} + \cos^4\alpha + \cancel{\cos^2\alpha} = \frac{4}{5} \rightarrow \cos^4\alpha + \sin^2\alpha = \frac{4}{5}$

۲ ✓

۵

$\sin\alpha\tan\alpha - \frac{1}{\cos\alpha} > 0 \rightarrow \frac{\sin^2\alpha - 1}{\cos\alpha} > 0 \rightarrow \frac{-\cos^2\alpha}{\cos\alpha} > 0 \rightarrow \cos\alpha < 0$

$\frac{3\sin\alpha + \sin\alpha\cos\alpha}{\frac{2(1-\cos^2\alpha)}{\sin^2\alpha}} < 0 \rightarrow \frac{\overbrace{3 + \cos\alpha}^{\text{همواره} +}}{2\sin\alpha} < 0 \rightarrow \sin\alpha < 0$

$\Big\}$ α در ربع ۳ قرار دارد

۲ ✓

$\dfrac{2\cos\alpha + 3\sin\alpha}{\sin\alpha\cos\alpha} = 0 \to 2\cos\alpha + 3\sin\alpha = 0 \to 2\cos\alpha = -3\sin\alpha \to \tan\alpha = \dfrac{-2}{3} \to$ $\cot\alpha = \dfrac{-3}{2}$ ۲ $\tan\alpha + \cot\alpha = \dfrac{-2}{3} + \dfrac{-3}{2} = \boxed{\dfrac{-13}{6}}$	۶
$\dfrac{\sqrt{1+\sin 80}}{\sin 50 + \sin 10} = \dfrac{\sqrt{\sin^2 40 + \cos^2 40 + 2\sin 40\cos 40}}{2\sin\frac{50+10}{2}\cos\frac{50-10}{2}} = \dfrac{\sin 40 + \cos 40}{\cancel{2}\cancel{\sin 30}\cos 20} = \dfrac{\sqrt{2}\sin(\overbrace{25+45}^{70})}{\cos 20} =$ $\dfrac{\sqrt{2}\cos 20}{\cos 20} = \boxed{\sqrt{2}}$ ۲	۷
$(1-2\sin^2 5)(2\cos^2 10 - 1)\left(\dfrac{1-\tan^2 20}{1+\tan^2 20}\right) = (\cos 10)(\cos 20)(\cos 40) \times \dfrac{\sin 10}{\sin 10}$ $\underbrace{(\cos 10)(2\cos^2 10 - 1)\left(2(2\cos^2 10 - 1)^2 - 1\right)}$ $\dfrac{1}{8}\,\dfrac{\cos 10}{\sin 10} = \dfrac{1}{8}\cot 10$ ۱,۵	۸
$(\sin\alpha + 3\cos\alpha)^2 = 4 \to \sin^2\alpha + 6\sin\alpha\cos\alpha + 9\cos^2\alpha = 4 \to 8\cos^2\alpha + 6\sin\alpha\cos\alpha = 3 \to$ $\xrightarrow{(\sin\alpha = 2-3\cos\alpha)} 8\cos^2\alpha + 6\cos\alpha(2-3\cos\alpha) = 3 \to -10\cos^2\alpha + 12\cos\alpha - 3 = 0 \to$ $10\cos^2\alpha - 12\cos\alpha + 3 = 0 \to 10\cos^2\alpha - 12\cos\alpha = -3$ $5(2\cos^2\alpha - 1) - 12\cos\alpha = \underbrace{10\cos^2\alpha - 12\cos\alpha}_{-3} - 5 = \boxed{-8}$ ۲	۹
$\sin B\sin C = \cos^2\dfrac{A}{2} \xrightarrow{B=72} \sin 72\sin C = \cos^2\dfrac{A}{2} \xrightarrow{C=72 \text{ الزاماً}} \sin^2 72 = \cos^2\dfrac{A}{2} \to$ $\dfrac{A}{2} = 90 - 72 = 18 \to A = 36$ $\quad 36 + 72 + 72 = 180$ ✓ $\Rightarrow$ بنابراین $A = 36$ لزوماً ۲	۱۰