آیلا میرزینیا

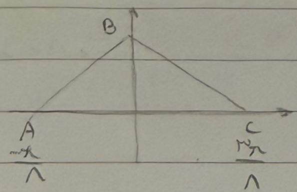
۱. $y=\tan(-2x+\frac{\pi}{4}) \to B = \tan(\frac{\pi}{4}) = 1 \to B = (0,1)$

$\to A = (\ ,\ \tan(-2x+\frac{\pi}{4}))$ تن

$\to \tan(-2x+\frac{\pi}{4}) = \tan(\frac{\pi}{2}) = -2x = \frac{\pi}{2}-\frac{\pi}{4} \to x = \frac{-\pi}{8}\ (A)$

$\to \tan(-2x+\frac{\pi}{4}) = \tan(-\frac{\pi}{2}) \to -2x = \frac{-\pi}{2}-\frac{\pi}{4} \to x = \frac{3\pi}{8}\ (C)$

$C-A = \frac{\pi}{2}$



$\to \frac{1\times(\frac{3\pi}{8}-(-\frac{\pi}{8}))}{2} = \frac{\pi}{4}$

۲. $x^2-(\sin\alpha)x-\frac{1}{4}\cos^2\alpha = 0 \xrightarrow{x=\frac{2}{3}} \frac{4}{9}-\frac{2}{3}\sin\alpha-\frac{1}{4}(1-\sin^2\alpha)=0$

$\xrightarrow{\times 36} 16-24\sin\alpha-9(1-\sin^2\alpha) = 9\sin^2\alpha-24\sin\alpha+7=0$

$\sin\alpha = \frac{24\pm\sqrt{576-252}}{18}$ ($\sqrt{576-252}=18$) $\to \frac{42}{18}$ غ ق ق

$\to \frac{6}{18}=\frac{1}{3} \to \cos\alpha = \sqrt{1-\sin^2\alpha} = \sqrt{1-\frac{1}{9}} = \frac{\sqrt{8}}{3}$

$1+\tan^2\alpha = \frac{1}{\cos^2\alpha} = \frac{9}{8}$

۳. $\log(\sin\alpha)-\log(-\cos\alpha) = \log 3 \to \log\frac{\sin\alpha}{-\cos\alpha} = \log 3 \to \sin\alpha = -3\cos\alpha$

$\sin^2\alpha+\cos^2\alpha=1 \to 9\cos^2\alpha+\cos^2\alpha=1 \to \cos^2\alpha=\frac{1}{10} \to \cos\alpha = \frac{-\sqrt{10}}{10}\ /\ \sin\alpha=\frac{3\sqrt{10}}{10}$

$\sin\alpha-\cos\alpha = \frac{3\sqrt{10}}{10}-(\frac{-\sqrt{10}}{10}) = \frac{4\sqrt{10}}{10} = \frac{2\sqrt{10}}{5}$

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۴. $\sin^4\theta+\cos^2\theta=\frac{4}{5} \to \sin^4\theta+1-\sin^2\theta=\frac{4}{5} \to \sin^2\theta(\sin^2\theta-1)=\frac{-1}{5}$

$\sin^2\theta\cos^2\theta=\frac{1}{5}$

$\cos^4\theta+\sin^2\theta=\cos^4\theta+1-\cos^2\theta=\cos^2\theta(\cos^2\theta-1)+1=-\sin^2\theta\cos^2\theta+1$

$=\frac{-1}{5}+1=\frac{4}{5}$

۵. $\frac{3\sin x+\sin x\cos x}{2-2\cos^2 x}<0 \to \frac{\sin x(\overbrace{3+\cos x})}{2(\underbrace{1-\cos^2 x}_{\sin^2 x})}=\frac{\overbrace{3+\cos x}^{\text{همواره مثبت}}}{2\sin x}<0$

$2\sin x<0 \to \sin x<0$

$\sin x\tan x-\frac{1}{\cos x}>0 \to \frac{1}{\cos x}(\underbrace{\cancel{\sin^2 x-1}}_{-\cos^2 x})>0 \to -\cos x>0 \to \cos x<0 \to$ ناحیه سوم

۶. $\frac{2}{\sin x}+\frac{3}{\cos x}=0 \to \frac{2\cos x+3\sin x}{\sin x\cos x}=0 \to 2\cos x+3\sin x=0$

$\cos x=\frac{-3}{2}\sin x$ / $\sin^2 x+\cos^2 x=1 \to \sin^2 x+\frac{9}{4}\sin^2 x=1 \to \sin^2 x=\frac{4}{13}$

$\to \sin x=\pm\frac{2\sqrt{13}}{13}$ $\quad \cos x=\frac{3\sqrt{13}}{13}$ عکس علامت $\sin x\cos x<0$

$\tan x+\cot x=\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}=\frac{\sin^2 x+\cos^2 x}{\sin x\cos x}=\frac{1}{\frac{-2\sqrt{13}}{13}\times\frac{3\sqrt{13}}{13}}=\frac{-13}{6}$

۷. $1+\sin 50=1+\sin 25\cos 25=(\sin 25+\cos 25)^2 \to \sqrt{1+\sin 50}=\sin 25\cos 25$

$\sin\alpha+\cos\alpha=\sqrt{2}\sin(\alpha+\frac{\pi}{4}) \to \sin 25+\cos 25=\sqrt{2}\sin 70=\sqrt{2}\cos 20$

$\sin 50+\sin 10=\sin(20+30)+\sin(30-20)=\cancel{\sin 20\cos 30}+\sin 30\cos 20+\sin 30\cos 20-\cancel{\cos 30\sin 20}$

$=2\sin 30\cos 20=\cos 20$

$\to \frac{\sqrt{1+\sin 50}}{\sin 50+\sin 10}=\frac{\sqrt{2}\cos 20}{\cos 20}=\sqrt{2}$

آیلا فرزانه

۸.

$\sin^2\alpha = \frac{1-\cos2\alpha}{2} \to \cos2\alpha = 1-2\sin^2\alpha \to 1-2\sin^2 20 = \cos40$

$\cos^2\alpha = \frac{1+\cos2\alpha}{2} \to \cos2\alpha = 2\cos^2\alpha - 1 \to 2\cos^2 10 - 1 = \cos20$

$\cos2\alpha = \frac{1-\tan^2\alpha}{1+\tan^2\alpha} \to \frac{1-\tan^2 20}{1+\tan^2 20} = \cos40$

$\frac{1}{2}\sin20 \quad \frac{1}{4}\sin40 \quad \frac{1}{8}\sin80$

$\to (1-2\sin^2 20)(2\cos^2 10 - 1)\left(\frac{1-\tan^2 20}{1+\tan^2 20}\right) = \frac{\sin10}{\sin10} \times \cos10 \times \cos20 \times \cos40 = \frac{1}{8}\frac{\sin80}{\sin10}$

$\to \frac{1}{8}\frac{\sin(\frac{\pi}{2}-10)}{\sin10} = \frac{1}{8}\frac{\cos10}{\sin10} = \frac{1}{8}\cot10$

۹.

$\cos2\alpha = 2\cos^2\alpha - 1 \to 5\cos2\alpha - 12\cos\alpha = 10\cos^2\alpha - 12\cos\alpha - 5 = ?$

$\sin\alpha + 3\cos\alpha = 2 \to \sin\alpha = 2 - 3\cos\alpha \to \sin^2\alpha + \cos^2\alpha = 1$

$(2-3\cos\alpha)^2 + \cos^2\alpha = 9\cos^2\alpha + 4 - 12\cos\alpha + \cos^2\alpha = 10\cos^2\alpha - 12\cos\alpha + 4 = 1$

$10\cos^2\alpha - 12\cos\alpha = -3$

$\to 10\cos^2\alpha - 12\cos\alpha - 5 = -3 - 5 = \underline{\underline{-8}}$

۱۰.

$\cos^2\frac{A}{2} = \frac{1+\cos A}{2}$

$\cos A = \cos(\pi - (B+C)) = -\cos(B+C) \to 1 - \cos(B+C) = 2\sin B\sin C$

$1 - (\cos B\cos C - \sin B\sin C) = 2\sin B\sin C$

$1 - \cos B\cos C + \sin B\sin C = 2\sin B\sin C$

$1 - (\cos B\cos C + \sin B\sin C) = 0 \to 1 - \cos(B-C) = 0 \to \cos(B-C) = 1 \to B - C = 0$

$\to B = C = 72$

$A = 180 - (B+C) = 180 - 144 = 36$