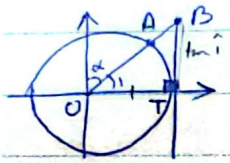
① $AB = ?$ ← بر حسب $\sin\alpha$



$1^2 + \tan^2 O_1 = OB^2 \quad \frac{1}{\cos^2 O_1} = OB^2 \rightarrow \frac{1}{\cos O_1} = OB$

$O_1 + \alpha = 90 \rightarrow \frac{1}{\sin\alpha} = OB$

$\rightarrow AB = OB - OA = \frac{1}{\sin\alpha} - 1 = \frac{1-\sin\alpha}{\sin\alpha}$

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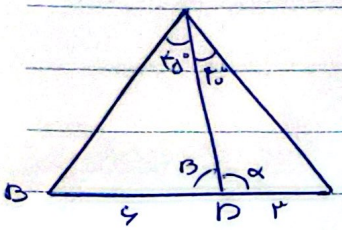
② $\frac{AB}{AC} = ?$



$\triangle ABD: \frac{6}{\sin 45} = \frac{AB}{\sin B} \qquad \triangle ACD: \frac{2}{\sin 30} = \frac{AC}{\sin\alpha}$

$\rightarrow \alpha + B = 180 \rightarrow \sin\alpha = \sin B$ (*)

$\sin B = \frac{AB \times \sin 45}{6}, \quad \sin\alpha = \frac{AC \times \sin 30}{2}$ (*) $\rightarrow$

$\frac{AB \times \frac{\sqrt{2}}{2}}{6} = \frac{AC \times \frac{1}{2}}{2} \rightarrow \frac{AB}{AC} = \frac{12}{6\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

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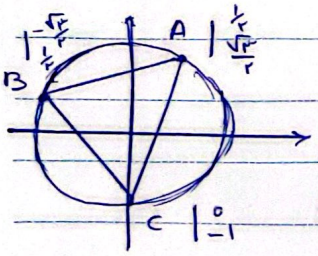
③ $A\left|\frac{1}{2}\right.$ $B\left|\frac{?}{\frac{1}{2}}\right.$



الف) محیط مثلث؟ $AB = \sqrt{\left(\frac{1}{2} + \frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2} - \frac{1}{2}\right)^2} = \sqrt{\left(\frac{1}{4} + \frac{2}{4} + \frac{\sqrt{2}}{4}\right) + \left(\frac{1}{4} + \frac{2}{4} - \frac{\sqrt{2}}{4}\right)}$

$\rightarrow AB = \sqrt{2}$

$AC = \sqrt{\left(\frac{1}{2} - 0\right)^2 + \left(\frac{\sqrt{2}}{2} + 1\right)^2} = \sqrt{\frac{1}{4} + \frac{2}{4} + 1 + \sqrt{2}} = \sqrt{2+\sqrt{2}}$

$BC = \sqrt{\left(0 + \frac{\sqrt{2}}{2}\right)^2 + \left(-1 - \frac{1}{2}\right)^2} = \sqrt{\frac{2}{4} + \frac{9}{4}} = \sqrt{3}$

$S = \frac{abc}{4r} = \frac{(\sqrt{2})(\sqrt{2})(\sqrt{2+\sqrt{2}})}{4} = \frac{(\sqrt{6})(\sqrt{2+\sqrt{2}})}{4} = \frac{\sqrt{12+6\sqrt{2}}}{4}$

$= \frac{\sqrt{(3+\sqrt{2})^2}}{4} = \frac{3+\sqrt{2}}{4}$

ب) محیط $= \sqrt{2} + \sqrt{2+\sqrt{2}} + \sqrt{3}$

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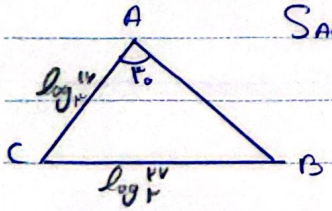
④ $AC = \log_3^{16} \qquad BC = \log_2^{27}$

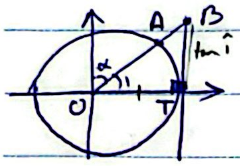


$S_{ABC} = \frac{1}{2} \times AB \times AC \times \sin A = \frac{1}{2} \times$

$\frac{\log^{27}_2}{\frac{1}{2}} = \frac{\log^{16}_3}{\sin B} \rightarrow \sin B = \frac{\frac{1}{2}\log^{16}_3}{\log^{27}_2} = \frac{\frac{1}{2} \times 4 \log^2_3}{3\log^2_3} = \frac{2}{3} \times \log^3_3 \times \log^2_2$

روزین قربانیان تکلیف شماره ۱۴

① $AB = ?$ $\longrightarrow$ برحسب $\sin\alpha$



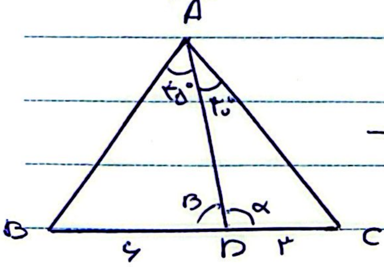
$1^2 + \tan^2 O_1 = OB^2$ $\quad \frac{1}{\cos^2 O_1} = OB^2 \longrightarrow \frac{1}{\cos O_1} = OB$

$O_1 + \alpha = 90 \longrightarrow \frac{1}{\sin\alpha} = OB$

$\longrightarrow AB = OB - OA = \frac{1}{\sin\alpha} - 1 = \frac{1-\sin\alpha}{\sin\alpha}$

---

② $\frac{AB}{AC} = ?$



$\overset{\triangle}{ABD}: \frac{6}{\sin 45} = \frac{AB}{\sin B}$ $\qquad \overset{\triangle}{ACD} = \frac{3}{\sin 30} = \frac{AC}{\sin\alpha}$

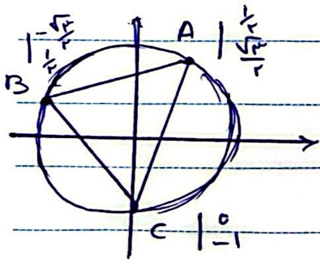
$\longrightarrow \alpha + B = 180 \longrightarrow \sin\alpha = \sin B$ (*)

$\sin B = \frac{AB \times \sin 45}{6}$, $\sin\alpha = \frac{AC\times \sin 30}{3}$ (*) طبق $\longrightarrow$

$\frac{AB\times\frac{\sqrt2}{2}}{6} = \frac{AC\times\frac12}{3} \longrightarrow \frac{AB}{AC} = \frac{6}{3\sqrt2} = \frac{2\sqrt2}{2} = \sqrt2$

---

③ $A\left|\begin{smallmatrix}\frac12\\ ?\end{smallmatrix}\right.$ $\quad B\left|\begin{smallmatrix}?\\ \frac12\end{smallmatrix}\right.$



الف) مساحت مثلث؟ $AB = \sqrt{(\frac12+\frac{\sqrt3}{2})^2 + (\frac{\sqrt3}{2}-\frac12)^2} = \sqrt{(\frac14+\frac34+\frac{\sqrt3}{2})+(\frac14+\frac34-\frac{\sqrt3}{2})}$

$\longrightarrow AB = \sqrt2$

$AC = \sqrt{(\frac12-0)^2+(\frac{\sqrt3}{2}+1)^2} = \sqrt{\frac14+\frac34+1+\sqrt3} = \sqrt{2+\sqrt3}$

$BC = \sqrt{(0+\frac{\sqrt3}{2})^2+(-1-\frac12)^2} = \sqrt{\frac34+\frac94} = \sqrt3$

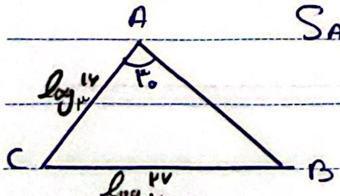
$S = \frac{abc}{4R} = \frac{(\sqrt3)(\sqrt2)(\sqrt{2+\sqrt3})}{4} = \frac{(\sqrt6)(\sqrt{2+\sqrt3})}{4} = \frac{\sqrt{12+6\sqrt3}}{4}$

$= \frac{\sqrt{(3+\sqrt3)^2}}{4} = \frac{3+\sqrt3}{4}$

ب) محیط $= \sqrt2 + \sqrt{2+\sqrt3} + \sqrt3$

---

④ $AC = \log_3^{16}$ $\quad BC = \log_2^{27}$



$S_{ABC} = \frac12 \times AB \times AC \times \sin 30 . \frac12 = \frac14 \times$

$\frac{\log_2^{27}}{\frac12} = \frac{\log_3^{16}}{\sin B} \longrightarrow \sin B = \frac{\frac12\log_3^{16}}{\log_2^{27}} = \frac{\frac12\times 4\log_3^2}{3\log_2^3} = \frac23 \times \log_3^2 \times \log_3^2$

④ $2x^4 - x^2 - \sin^2\alpha(2\sin^2\alpha - 1) = (x - \cos\alpha)(Q(x) + 1$

$2x^4 - x^2 - \sin^2\alpha(\cos 2\alpha) = (x - \cos\alpha)Q(x) + 1$

$x = \cos\alpha \longrightarrow 2\cos^4\alpha - \cos^2\alpha - \sin^2\alpha(2\sin^2\alpha - 1) = 1$

$2\cos^4\alpha - \cos^2\alpha - 2\sin^4\alpha - \sin^2\alpha = 1$

$2(\cos^4\alpha - \sin^4\alpha) - (\cos^2\alpha + \sin^2\alpha) = 1 \longrightarrow \cos^4\alpha - \sin^4\alpha = 1$

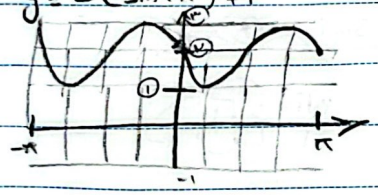
$(\cos^2\alpha + \sin^2\alpha)(\cos^2\alpha - \sin^2\alpha) = 1 \longrightarrow \cos 2\alpha = 1 \quad \cos^2 2\alpha = 1$

$\longrightarrow 1 - \cos^2 2\alpha = 0 \longrightarrow \sin^2 2\alpha = 0 \longrightarrow \boxed{\sin 2\alpha = 0}$

---

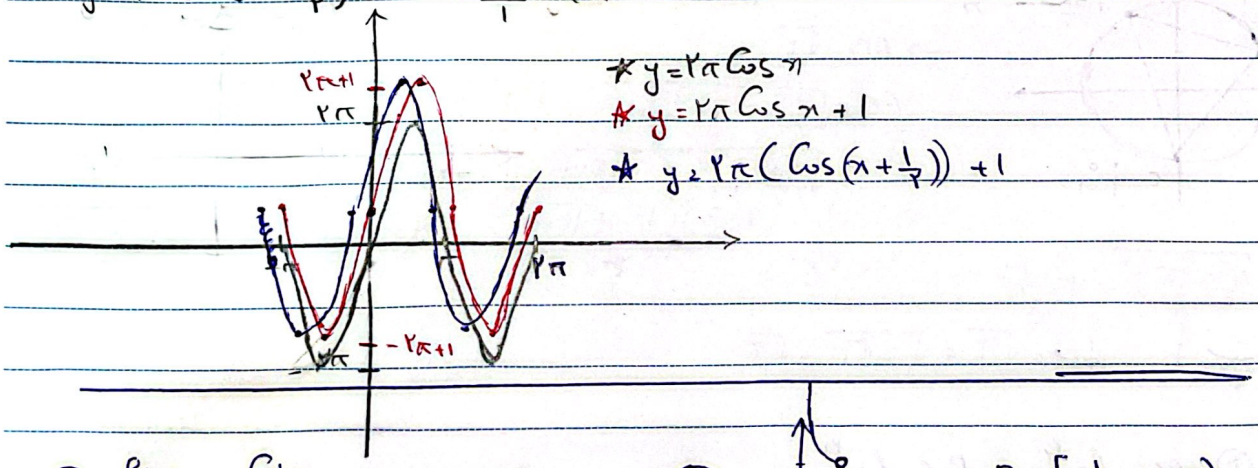
⑥ ا) $y = -2\sin x \cos x + 2 \quad [-\pi \ \ \pi]$

$y = -(\sin 2x) + 2 \qquad T = \frac{2\pi}{2} = \pi$



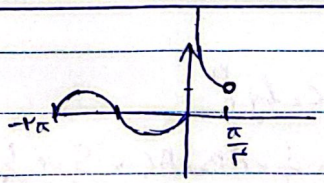
ب) $y = 2\cos(x + \frac{1}{2})\pi + 1 \quad [-\pi \ \ 2\pi]$

$y = 2\pi\cos(x + \frac{1}{2}) + 1 \qquad \frac{2\pi}{1} = 2\pi$



* $y = 2\pi\cos x$
* $y = 2\pi\cos x + 1$
* $y = 2\pi(\cos(x + \frac{1}{2})) + 1$

⑦ $f(x) = \begin{cases} \cot x & ; 0 < x < \frac{\pi}{2} \\ \sin x & x \le 0 \end{cases}$



$R = [-1 \ +\infty)$

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⑧ $f(x) = a + b\sin(cx)\cos(cx)\cos(2cx)$ (~~$\frac{1}{2}\sin(2cx)$~~ ~~$\frac{1}{2}\sin(4cx)$~~)

$f(x) = a + \frac{b}{4}\sin(4cx) \qquad T = \frac{\pi}{2} - \frac{\pi}{10} = \frac{4\pi}{10} = \frac{2\pi}{5} \longrightarrow \frac{2\pi}{|4c|} = \frac{2\pi}{5}$

$\frac{\max + \min}{2} = a \longrightarrow a = \frac{0+4}{2} = 2 \qquad \longrightarrow c = \frac{5}{4}$

$f(\frac{\pi}{4}) = 4 \longrightarrow 2 + \frac{b}{4}\sin(\frac{5\pi}{2}) = 4 \longrightarrow \frac{b}{4} = 2 \quad \boxed{b = 8}$

$\frac{bc}{a} = \boxed{\frac{5}{2}}$

۹ - $f(x) = a\cos bx + c$ $\quad T = \pi$ $\quad \frac{2\pi}{|b|} = \pi \rightarrow b = \pm 2$

$c = \frac{\max + \min}{2} = \frac{-2}{2} = -1$ $\rightarrow a = 2$ $\rightarrow f(x) = -2\cos 2x - 1$

$f(\alpha) = -2\cos 2\alpha - 1 = 0$ $\quad \cos 2\alpha = -\frac{1}{2}$ $\quad 1 + \tan^2 2\alpha = \frac{1}{\cos^2 2\alpha}$ $\quad \tan^2 2\alpha =$

$\tan^2 2\alpha = 8 \rightarrow \tan 2\alpha = \boxed{2\sqrt{2}}$

---

۱۰ - $y = f(x)$ $\quad f(-2210) = ?$ $\quad f(-2210 + 12T) = f(110)$

$f(x) = \begin{cases} 0.10x + 1 & 1 \le x < 2 \\ 2x + 2 & -1 \le x < 1 \end{cases}$ $\rightarrow f(110) = -0.10(110) + 1 = \boxed{0.125}$