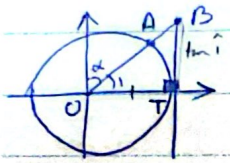
① $AB = ?$ → $\sin\alpha$ برحسب



$1^2 + \tan^2 O_1 = OB^2 \quad \frac{1}{\cos^2 O_1} = OB^2 \rightarrow \frac{1}{\cos O_1} = OB$

$O_1 + \alpha = 90 \rightarrow \frac{1}{\sin\alpha} = OB$

$\rightarrow AB = OB - OA = \frac{1}{\sin\alpha} - 1 = \frac{1-\sin\alpha}{\sin\alpha}$ ✓ (۲)

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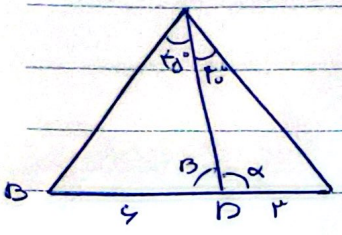
② $\frac{AB}{AC} = ?$



$\triangle ABD: \frac{6}{\sin 45} = \frac{AB}{\sin B} \qquad \triangle ACD: \frac{2}{\sin 30} = \frac{AC}{\sin\alpha}$

$\rightarrow \alpha + B = 180 \rightarrow \sin\alpha = \sin B$ (*)

$\sin B = \frac{AB \times \sin 45}{6}, \quad \sin\alpha = \frac{AC \times \sin 30}{2}$ (*) $\rightarrow$

$\frac{AB \times \frac{\sqrt{2}}{2}}{6} = \frac{AC \times \frac{1}{2}}{2} \rightarrow \frac{AB}{AC} = \frac{2 \times 6}{6\sqrt{2}} = \frac{12\sqrt{2}}{2}$ ✓ (۲)

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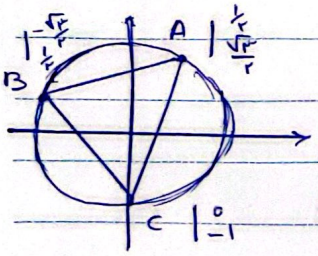
③ $A\left|\frac{1}{2}\right.$ $B\left|\frac{?}{\frac{1}{2}}\right.$



الف) طول وتر؟ $AB = \sqrt{(\frac{1}{2}+\frac{\sqrt{2}}{2})^2 + (\frac{\sqrt{2}}{2}-\frac{1}{2})^2} = \sqrt{(\frac{1}{4}+\frac{2}{4}+\frac{\sqrt{4}}{4}) + (\frac{1}{4}+\frac{2}{4}-\frac{\sqrt{4}}{4})}$

$\rightarrow AB = \sqrt{2}$

$AC = \sqrt{(\frac{1}{2}-0)^2 + (\frac{\sqrt{2}}{2}+1)^2} = \sqrt{\frac{1}{4}+\frac{2}{4}+1+\sqrt{2}} = \sqrt{2+\sqrt{2}}$

$BC = \sqrt{(0+\frac{\sqrt{2}}{2})^2 + (-1-\frac{1}{2})^2} = \sqrt{\frac{2}{4}+\frac{9}{4}} = \sqrt{3}$

$S = \frac{abc}{4r} = \frac{(\sqrt{2})(\sqrt{2})(\sqrt{2+\sqrt{2}})}{4} = \frac{(\sqrt{6})(\sqrt{2+\sqrt{2}})}{4} = \frac{\sqrt{12+6\sqrt{2}}}{4}$

$= \frac{\sqrt{(3+\sqrt{2})^2}}{4} = \frac{3+\sqrt{2}}{4}$

ب) محیط $= \sqrt{2} + \sqrt{2+\sqrt{2}} + \sqrt{2}$ ✓ (۱٫۷۵)

$S = \frac{1}{2} + \frac{1}{4} + \frac{\sqrt{3}}{4} = \frac{3+\sqrt{3}}{4}$

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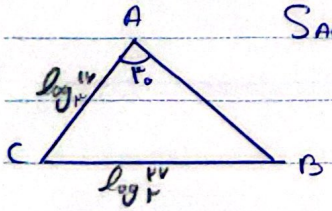
④ $AC = \log_3^{16} \qquad BC = \log_2^{27}$ (۲)



$S_{ABC} = \frac{1}{2} \times AB \times AC \times \sin 30 = \frac{1}{2} \times$

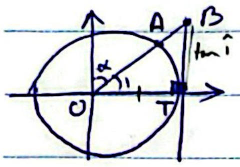
$\frac{\log^{27}_2}{\frac{1}{2}} = \frac{\log^{16}_2}{\sin B} \rightarrow \sin B = \frac{\frac{1}{2}\log^{16}_2}{\log^{27}_2} = \frac{\frac{1}{2} \times 4 \log^2_2}{3\log^2_3} = \frac{2}{3} \times \log^2_3 \times \log^2_2$

$S = \frac{1}{2} \times AB \times AC \times \sin 30 = \frac{1}{4} \times \log^{27}_2 \times \log^{16}_3$

$S = \frac{1}{4} \times 3 \times 4 \times \log^3_2 \times \log^2_3 = 3$

رودین قربانیان تکلیف شماره ۱۴

① $AB = ?$ → برحسب $\sin\alpha$



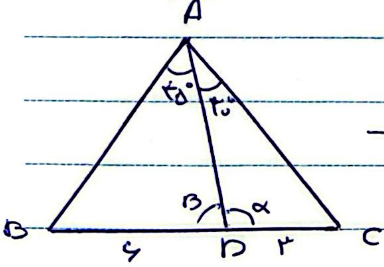
$1^2 + \tan^2 O_1 = OB^2 \qquad \frac{1}{\cos^2 O_1} = OB^2 \longrightarrow \frac{1}{\cos O_1} = OB$

$O_1 + \alpha = 90 \longrightarrow \frac{1}{\sin\alpha} = OB$

$\longrightarrow AB = OB - OA = \frac{1}{\sin\alpha} - 1 = \frac{1-\sin\alpha}{\sin\alpha}$

---

② $\frac{AB}{AC} = ?$



$\overset{\triangle}{ABD}: \frac{6}{\sin 45} = \frac{AB}{\sin B} \qquad \overset{\triangle}{ACD} = \frac{3}{\sin 30} = \frac{AC}{\sin\alpha}$

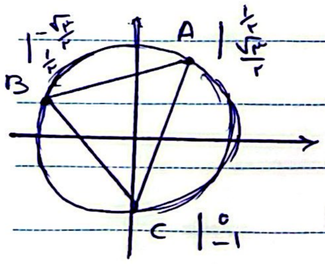
$\longrightarrow \alpha + B = 180 \longrightarrow \sin\alpha = \sin B$ (*)

$\sin B = \frac{AB \times \sin 45}{6}, \quad \sin\alpha = \frac{AC \times \sin 30}{3}$ طبق (*) ⟶

$\frac{AB \times \frac{\sqrt{2}}{2}}{6} = \frac{AC \times \frac{1}{2}}{3} \longrightarrow \frac{AB}{AC} = \frac{24}{12\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

---

③ $A \left| \begin{matrix} \frac{1}{2} \\ ? \end{matrix} \right. \qquad B \left| \begin{matrix} ? \\ \frac{1}{2} \end{matrix} \right.$



الف) مساحت مثلث؟ $AB = \sqrt{\left(\frac{1}{2}+\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)^2} = \sqrt{\left(\frac{1}{4}+\frac{3}{4}+\frac{\sqrt{3}}{2}\right)+\left(\frac{1}{4}+\frac{3}{4}-\frac{\sqrt{3}}{2}\right)}$

$\longrightarrow AB = \sqrt{2}$

$AC = \sqrt{\left(\frac{1}{2}-0\right)^2 + \left(\frac{\sqrt{3}}{2}+1\right)^2} = \sqrt{\frac{1}{4}+\frac{3}{4}+1+\sqrt{3}} = \sqrt{2+\sqrt{3}}$

$BC = \sqrt{\left(0+\frac{\sqrt{3}}{2}\right)^2 + \left(-1-\frac{1}{2}\right)^2} = \sqrt{\frac{3}{4}+\frac{9}{4}} = \sqrt{3}$

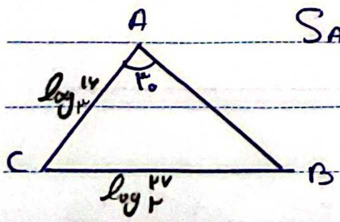
$S = \frac{abc}{4R} = \frac{(\sqrt{3})(\sqrt{2})(\sqrt{2+\sqrt{3}})}{4} = \frac{(\sqrt{6})(\sqrt{2+\sqrt{3}})}{4} = \frac{\sqrt{12+6\sqrt{3}}}{4}$

$= \frac{\sqrt{(3+\sqrt{3})^2}}{4} = \frac{3+\sqrt{3}}{4}$

ب) محیط $= \sqrt{2} + \sqrt{2+\sqrt{3}} + \sqrt{3}$

---

④ $AC = \log_3^{16} \qquad BC = \log_2^{27}$



$S_{ABC} = \frac{1}{2} \times AB \times AC \times \sin 30 \cdot \frac{1}{2} = \frac{1}{4} \times$

$\frac{\log_2^{27}}{\frac{1}{2}} = \frac{\log_3^{16}}{\sin B} \longrightarrow \sin B = \frac{\frac{1}{2}\log_3^{16}}{\log_2^{27}} = \frac{\frac{1}{2} \times 4 \log_3^{2}}{3\log_2^{3}} = \frac{2}{3} \times \log_3^{2} \times \log_3^{2}$

④ $2x^4 - x^2 - \sin^2\alpha(2\sin^2\alpha - 1) = (x - \cos\alpha)(Q(x) + 1$

$2x^4 - x^2 - \sin^2\alpha(\cos 2\alpha) = (x - \cos\alpha)Q(x) + 1$

$x = \cos\alpha \longrightarrow 2\cos^4\alpha - \cos^2\alpha - \sin^2\alpha(2\sin^2\alpha - 1) = 1$

$2\cos^4\alpha - \cos^2\alpha - 2\sin^4\alpha - \sin^2\alpha = 1$

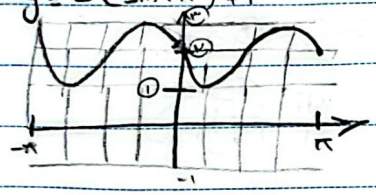
$2(\cos^4\alpha - \sin^4\alpha) - (\cos^2\alpha + \sin^2\alpha) = 1 \longrightarrow \cos^4\alpha - \sin^4\alpha = 1$

$(\cos^2\alpha + \sin^2\alpha)(\cos^2\alpha - \sin^2\alpha) = 1 \longrightarrow \cos 2\alpha = 1 \quad \cos^2 2\alpha = 1$

$\longrightarrow 1 - \cos^2 2\alpha = 0 \longrightarrow \sin^2 2\alpha = 0$ $\boxed{\sin 2\alpha = 0}$ ✓ (2)

⑤ الحل) $y = -2\sin x \cos x + 2$ $[-\pi \quad \pi]$

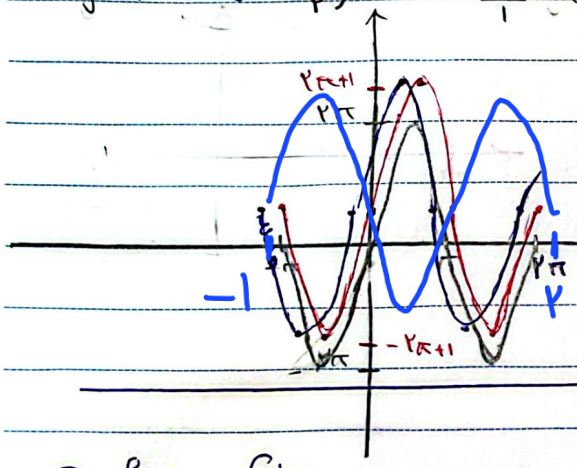
$y = -(\sin 2x) + 2$ $T = \frac{2\pi}{2} = \pi$ ✓



1,0

ب) $y = 2\cos(x + \frac{1}{2})\pi + 1$ $[-\pi \quad 2\pi]$

$y = 2\pi\cos(x + \frac{1}{2}) + 1$ $\frac{2\pi}{1} = 2\pi$ $T = \frac{2\pi}{\pi} = 2$

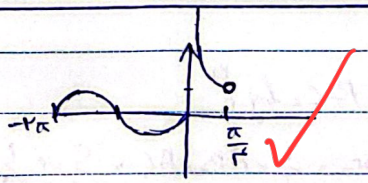


$\star\ y = 2\pi\cos x$

$\star\ y = 2\pi\cos x + 1$

$\star\ y = 2\pi(\cos(x + \frac{1}{2})) + 1$

⑥ $f(x) = \begin{cases} \cot x & ; 0 < x < \frac{\pi}{2} \\ \sin x & x \le 0 \end{cases}$



$R = [-1 \quad +\infty)$ ✓ (2)

⑦ $f(x) = a + b\sin(cx)\cos(cx)\cos(2cx)$ (~~$\frac{1}{2}\sin(2cx)$~~ ~~$\frac{1}{2}\sin(4cx)$~~)

$f(x) = a + \frac{b}{4}\sin(4cx)$ $T = \frac{\pi}{2} - \frac{\pi}{4} = \frac{4\pi}{10} = \frac{2\pi}{5} \longrightarrow \frac{2\pi}{|4c|} = \frac{2\pi}{5}$

$\frac{\max + \min}{2} = a \longrightarrow a = \frac{6+2}{2} = 4$ ✓ $\longrightarrow c = \frac{5}{4}$ ✓

$f(\frac{\pi}{4}) = 6 \longrightarrow 4 + \frac{b}{4}\sin(\frac{5\pi}{2}) = 6 \longrightarrow \frac{b}{4} = 2$ $\boxed{b = 8}$

$b = 8$

$\frac{bc}{a} = \boxed{\frac{a}{2}} \longrightarrow \frac{bc}{a} = \frac{a}{4}$

MODIR

1,5

۹ - $f(x) = a\cos bx + c \qquad T = \pi \qquad \frac{2\pi}{|b|} = \pi \rightarrow b = \pm 2$

$c = \frac{\max + \min}{2} = \frac{-2}{2} = -1 \longrightarrow a = 2 \longrightarrow f(x) = -2\cos 2x - 1$

$f(\alpha) = -2\cos 2\alpha - 1 = 0 \qquad \cos 2\alpha = -\frac{1}{2} \qquad 1 + \tan^2 2\alpha = \frac{1}{\cos^2 2\alpha} \qquad \tan^2 2\alpha =$

$\tan^2 2\alpha = 8 \longrightarrow \tan 2\alpha = \boxed{2\sqrt{2}}$ ✓ (۲)

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۱۰ - $y = f(x) \qquad f(-2240) = ? \qquad f(-224 + 112) = f(112)$

$f(x) = \begin{cases} 0.05x + 1 & 1 \le x < 2 \\ 2x + 2 & -1 \le x < 1 \end{cases} \longrightarrow f(112) = -0.05(112) + 1 = \boxed{0.4}$ ✓ (۲)