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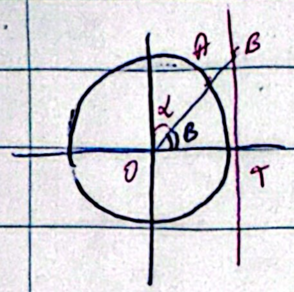
السیدضان محرک

①



$AO = 1$ شعاع دایره

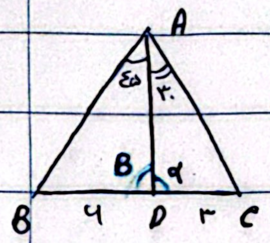
$OB^2 = 1 + \tan^2\beta = 1 + \cot^2\alpha \Rightarrow \frac{1}{\sin^2\alpha}$

$AB = OB - AO = \frac{1}{\sin^2\alpha} - 1$ → $AB = \frac{1}{\sin\alpha} - 1$

($OB = \frac{1}{\sin\alpha}$)

۱٫۵

②



$\alpha + \beta = 180 \Rightarrow \sin\alpha = \sin\beta$

$\frac{\sin 45}{2} = \frac{\sin\beta}{AB}$ $\quad$ $\frac{\sin 30}{3} = \frac{\sin\alpha}{AC}$

$\frac{\frac{1}{2}}{3} = \frac{\sqrt{2}AB}{2AC}$ $\quad$ $\frac{\frac{\sqrt{2}}{2}AB}{2}$ به جای $\sin\alpha$ می‌نویسیم

$\frac{AB}{AC} = \frac{2}{3\sqrt{2}} = \frac{\sqrt{2}}{3}$ ✓

۲

③

$A = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, $B = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$, $C = (0, -1)$

الف) مساحت مثلث؟

$$\left|\begin{matrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 1 \\ 0 & -1 & 1 \end{matrix}\right| \begin{matrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \\ 0 & -1 \end{matrix}$$

$\frac{1}{2}\left(\frac{1}{4} + 0 - \frac{\sqrt{3}}{2} - \left(0 - \frac{1}{2} + \frac{3}{4}\right)\right) = \frac{\sqrt{3}}{4}$

$S = \frac{3 + \sqrt{3}}{4}$

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ب)

$\overline{AB} = \sqrt{\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)^2 + \left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right)^2} = \sqrt{2 - 2\sqrt{3}}$ → $AB = \sqrt{2}$

$\frac{3}{4} + \frac{1}{4} - \sqrt{3}$

$\overline{AC} = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2} + 1\right)^2} = \sqrt{2 + \sqrt{3}}$ $\quad$ محیط $= \sqrt{2 - 2\sqrt{3}} + \sqrt{2 + \sqrt{3}} + \sqrt{3}$

$\frac{1}{4} + \frac{3}{4} + 1 + \sqrt{3}$

$\overline{BC} = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{3}{2}\right)^2} = \sqrt{3}$ $\quad$ محیط $= \sqrt{2} + \sqrt{3} + \sqrt{2 + \sqrt{3}}$

$\frac{3}{4}$ $\quad$ $\frac{9}{4}$

s.a.m

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$S_{ABC} = \frac{1}{2} AB \times AC \times \sin ? = \quad \frac{1}{2}$ $\qquad \log \frac{?}{?} = \frac{1}{\log \frac{?}{?}}$

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$\frac{\sin \hat{A}}{BC} = \frac{\sin \hat{B}}{AC}$ $\quad BC = \log_2 27, \; AC = \log_3 16$ $\qquad \sin B = \frac{\log_2 27}{2\log_3 16} = \frac{4 \log ?}{4 \log ?} = \frac{1}{2}$

$S_{ABC} = \frac{1}{4} \times \log_2 27 \times \log_3 16 \times \frac{1}{2} = \frac{1}{4} \times 3 \times 4 \times \log_2 3 \times \log_3 2 = 3$

۵

$2x^4 - x^2 - \sin^2(\alpha)(2\sin^2\alpha - 1) \Rightarrow 2\cos^4\alpha - \cos^2\alpha - 2\sin^4\alpha + \sin^2\alpha$

$2\cos^4\alpha - 2\sin^4\alpha - (\cos^2\alpha - \sin^2\alpha) = 1 \qquad \cos 2\alpha = 1 \Rightarrow \sin 2\alpha = 0$

$2\cos 2\alpha - \cos 2\alpha$

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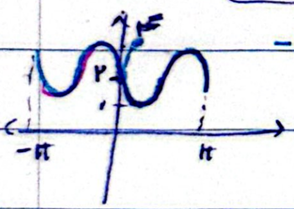
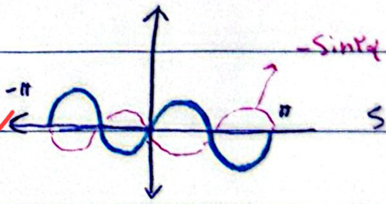
الف) $y = -2\sin\alpha\cos\alpha + 2$

$-\sin 2\alpha$

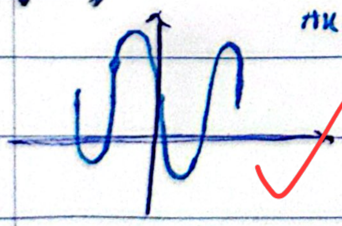


$T = \frac{2\pi}{2} = \pi$

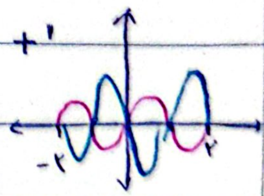


ب) $y = 2\cos(x + \frac{1}{2})\pi + 1 \Rightarrow y = -2\sin \pi x + 1$

$\pi x + \frac{\pi}{2}$



$T = \frac{2\pi}{\pi} = 2$



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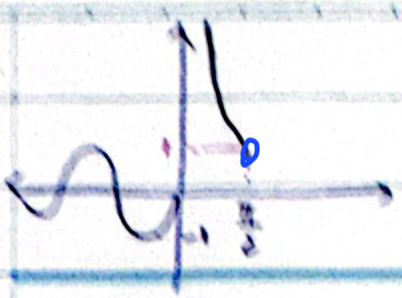

$R_f = (-1, +\infty)$ ✓ (۴)

(۷)

(۸) $f(x) = a + b\sin(cx)\cos(cx)\cos(cx) = a + \frac{b}{2}\sin(2cx)$

$\frac{1}{2}\sin(2cx)\cdot\cos(cx)$

$\frac{1}{2}\sin(2cx)\cdot\cos(cx)$

$\frac{2\pi}{4c} = \frac{2\pi}{\omega}$ ✓ $c = \frac{\omega}{4}$

$T = \frac{\pi}{\omega} = \frac{\pi}{10} = \frac{4\pi}{10} = \frac{2\pi}{5}$ (۴)

$a + |\frac{b}{2}| = 4 \quad 2a = 4 \quad a = 2$

$a - |\frac{b}{2}| = 0 \quad b = \pm 4$

$\frac{bc}{a} = \frac{8\sqrt{3}}{4\sqrt{2}}$ ✓ (۵)

(۹) $f(x) = a\cos bx + c \quad T = \pi = \frac{2\pi}{b} \rightarrow b = 2$

$|a| + c = 2 \Rightarrow 2c = -1 \quad c = -1$ (۱٫۵)

$c - |a| = -4 \quad a = \pm 3 \quad -2\sqrt{7} \quad f(x) = -2\cos 2x - 1 = 0$

$(\tan\alpha)^2 = 2 \Rightarrow \tan\alpha = \sqrt{2}$ $\quad +0.25 \quad \frac{1}{\cos 2x} - 1 \quad \cos 2x = -\frac{1}{3}$ ✓

$\tan 2\alpha = 2\sqrt{2}$ $\quad \tan 2\alpha$

(۱۰) $f(-2.4, 0) = f(+1, 0)$

$y = -\frac{1}{2}x + 1 \rightsquigarrow f(\xi) = -\frac{1}{3} + 1 = \frac{1}{4}$ ✓ (۴)