(پارسا امینی راد)

1

$y=(\tan\alpha)x$

$x_B=1 \Rightarrow y_B=\tan\alpha$ $\qquad B=(1,\tan\alpha)$

$AB=\sqrt{(1-\cos\alpha)^2+(\tan\alpha-\sin\alpha)^2} \Rightarrow \sqrt{(1-\cos\alpha)^2+(\frac{\sin\alpha}{\cos\alpha}-\sin\alpha)^2}$

$\to AB=(1-\cos\alpha)\sqrt{1+\frac{\sin^2\alpha}{\cos^2\alpha}}$

$1+\tan^2\alpha=\frac{1}{\cos^2\alpha}$ $\Big\} \to AB=\frac{1-\cos\alpha}{\cos\alpha}$ $\qquad \cos\alpha=\sqrt{1-\sin^2\alpha}$

$AB=\frac{1-\sqrt{1-\sin^2\alpha}}{\sqrt{1-\sin^2\alpha}} \longleftarrow \sin \xleftarrow{\text{برحسب}}$

2

$\frac{AB}{\sin(A\hat{O}B)}=\frac{BD}{\sin 45}$ $\qquad A\hat{O}B+A\hat{O}C=180 \xrightarrow{\text{مکمل}}$ sin برابر

$\frac{AC}{\sin(A\hat{O}C)}=\frac{DC}{\sin 30}$ $\longrightarrow \frac{AB}{AC}=\frac{BD\sin 45}{DC\sin 30} \to \frac{AB}{AC}=\frac{4\times\frac{\sqrt2}{2}}{3\times\frac12}=2\sqrt2$

3

$x_A=\frac12$, $y_B=\frac12 \to x^2+y^2=1 \to y_A=\sqrt{1-x_A^2}=\sqrt{1-(\frac12)^2}=\frac{\sqrt3}{2}$

$A(\frac12,\frac{\sqrt3}{2})$ / $x_B^2+y_B^2=1 \to x_B^2=1-\frac14=\frac34 \to x_B=\pm\frac{\sqrt3}{2}$

$B(-\frac{\sqrt3}{2},\frac12)$ $\qquad C(0,-1)$ $\qquad S=\frac12|x_A(y_B-y_C)+x_B(y_C-y_A)+x_C(y_A-y_B)|$

$S=\frac12|\frac12(\frac12+1)+-\frac{\sqrt3}{2}(-1-\frac{\sqrt3}{2})+0(\frac{\sqrt3}{2}-\frac12)|=\frac{3+\sqrt3}{4}$ الف)

$AB=\sqrt{(x_A-x_B)^2+(y_A-y_B)^2}=\sqrt{(\frac12+\frac{\sqrt3}{2})^2+(\frac{\sqrt3}{2}-\frac12)^2}=\sqrt2$ ب)

$AC=\sqrt{(\frac12)^2+(\frac{\sqrt3}{2}+1)^2}=\sqrt{2+\sqrt3}$ $\qquad BC=\sqrt{\frac34+\frac94}=\sqrt3$ $\qquad P=\sqrt2+\sqrt3+\sqrt{2+\sqrt3}$

4

$S=\frac12\times AB\times BC\times\sin A \to \frac12\times\log_3^{16}\times\log_2^{27}\times\underbrace{\sin\theta^{30}}_{\frac12}=\frac14\times\log_3^{16}\times\log_2^{27}$

$\log_3^{16}=4\log_3^2$ $\qquad \log_2^{27}=3\log_2^3$ $\qquad \to S=3\log_3^2\times\log_2^3$

$S\to 3\times\frac{1}{\log_2^3}\times\log_2^3=3\to$ SAB

(سانا امینی راد)

۵

$f(x)=2x^4-x^2-\sin^2\alpha(2\sin^2\alpha-1)$

$(x-\cos\alpha)=1$

$f(\cos\alpha)=1$

$\to 2(\cos\alpha)^4-(\cos\alpha)^2-\sin^2\alpha(2\sin^2\alpha-1)$

$2\cos^4\alpha-\cos^2\alpha-2\sin^4\alpha+\sin^2\alpha=1 \longrightarrow \sin^2\alpha=1-\cos^2\alpha$

$\to 2\cos^4\alpha-\cos^2\alpha-2(1-\cos^2\alpha)^2+(1-\cos^2\alpha)=1$

$\to 2\cos^4\alpha-\cos^2\alpha-2+4\cos^2\alpha-2\cos^4\alpha+1-\cos^2\alpha=1$

$(2\cos^4-2\cos^4)=0 \longrightarrow 2\cos^2\alpha-1=1 \longrightarrow \cos^2\alpha=1 \to \cos\alpha=\pm 1$

$\cos\alpha=1 \longrightarrow \sin\alpha=0$

$\cos\alpha=-1 \longrightarrow \sin\alpha=0 \to \sin 2\alpha=0 \to \sin 2\alpha=0$

الف

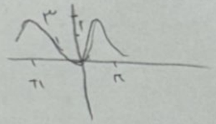
۶

$y=-2\sin x\cos x+2 \to x\in[-\pi,\pi] \longrightarrow \sin(2x)=2\sin x\cos x \to \frac{1}{2}\sin(2x)$

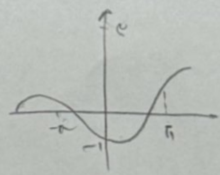
$\to y=-2\times\frac{1}{2}\sin(2x)+2=-\sin(2x)+2 \to$ دوره تناوب $\frac{2\pi}{2}=\pi$

ب) $T=\frac{2\pi}{1}$

د) شکل الف



شکل ب



۷

۸

$f(x)=4 \to x=\frac{\pi}{10}$

$f(x)=0 \to x=\frac{\pi}{2}$

$\sin(cx)\cos(cx)=\frac{1}{2}\sin(2cx)$

$\to a+\frac{b}{2}\sin(2cx)\cos(2cx)$

$\to \sin(2cx)\cos(2cx)=\frac{1}{4}\sin(4cx)$

$\to f(x)=a+\frac{b}{4}\sin(4cx) \longrightarrow A=\frac{b}{4}$ $\quad f_{max}=4$, $f_{min}=0$ $\quad a=\frac{4+0}{2}=2$

$\frac{b}{4}=2 \to b=8$ $\quad A=\frac{4-0}{2}=2$

$\frac{\pi}{2}-\frac{\pi}{10}=\frac{4\pi}{10}=\frac{2\pi}{5} \to \frac{T}{2}=\frac{2\pi}{5} \to T=\frac{4\pi}{5} \to \frac{\pi}{2c}=\frac{4\pi}{5} \to c=\frac{5}{8} \to \frac{bc}{a}=\frac{8(\frac{5}{8})}{2}=\frac{5}{2}$

(سارا امینی راد)

9

$a = \frac{2-(-4)}{2} = 3 \qquad c = \frac{2-4}{2} = -1$

$T = \frac{2\pi}{b} = \pi \longrightarrow b = 2$

$f(x) = 3\cos(2x) - 1 = 0 \longrightarrow \cos(2x) = \frac{1}{3} \longrightarrow x = \alpha \rightarrow 2\alpha = \arccos\frac{1}{3}$

$|\tan(2\alpha)| = 2\sqrt{2}$

$\hookrightarrow \frac{\sin(2\alpha)}{\cos(2\alpha)}$

10

$f(-34{,}5)? \longrightarrow f \quad (-1,-1) \rightarrow (0,2) \rightarrow (2,0)$

$-34{,}5 + 3n \in [-1,2]$

$-1 \leq -34{,}5 + 3n \leq 2 \rightarrow 11{,}16 \leq n \leq 12{,}16 \longrightarrow n = 12$

$x' = -34{,}5 + 3(12) = 1{,}5 \longrightarrow f(-34{,}5) = f(1{,}5)$

$(0,2) \quad (2,0) \longrightarrow m = \frac{0-2}{2-0} = -1 \longrightarrow f(x) = 2 - x \longrightarrow 0 \leq x \leq 2$

$f(1{,}5) = 2 - 1{,}5 = 0{,}5$