(پرستا امینی راد) ۱۲٫۵

①

$y = (\tan\alpha)\, x$

$x_B = 1 \Rightarrow y_B = \tan\alpha \qquad B = (1, \tan\alpha)$

$AB = \sqrt{(1-\cos\alpha)^2 + (\tan\alpha - \sin\alpha)^2} \Rightarrow \sqrt{(1-\cos\alpha)^2 + \left(\frac{\sin\alpha}{\cos\alpha} - \sin\alpha\right)^2}$

$\rightarrow AB = (1-\cos\alpha)\sqrt{1 + \frac{\sin^2\alpha}{\cos^2\alpha}}$

$1 + \tan^2\alpha = \cancel{\frac{1}{\cos^2\alpha}} \quad \frac{1}{\sin^2\alpha}$ $\Big\} \rightarrow AB = \frac{1-\cos\alpha}{\cos\alpha} \qquad \cos\alpha = \sqrt{1-\sin^2\alpha}$

$AB = \frac{1}{\sin\alpha} - 1$

$AB = \frac{1 - \sqrt{1-\sin^2\alpha}}{\sqrt{1-\sin^2\alpha}} \leftarrow$ sin برحسب $\leftarrow$

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②

$\frac{AB}{\sin(A\hat{O}B)} = \frac{BD}{\sin 45} \qquad A\hat{O}B + A\hat{O}C = 180 \rightarrow$ sin برابر

$\frac{AB}{AC} = \frac{4\sqrt{2}\sin\alpha}{4\sin\alpha} = \boxed{\sqrt{2}}$

$\frac{AC}{\sin(A\hat{O}C)} = \frac{DC}{\sin 30} \qquad AC = 4\sin\alpha \qquad AB = 4\sqrt{2}\sin\alpha$

$\longrightarrow \frac{AB}{AC} = \frac{BD\sin 45}{DC\sin 30} \rightarrow \frac{AB}{AC} = \frac{4\times\frac{\sqrt{2}}{2}}{3\times\frac{1}{2}} = 2\sqrt{2}$

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③

$x_A = \frac{1}{2} \;,\; y_B = \frac{1}{2} \rightarrow x^2 + y^2 = 1 \rightarrow y_A = \sqrt{1 - x_A^2} = \sqrt{1 - (\frac{1}{2})^2} = \frac{\sqrt{3}}{2}$

$A(\frac{1}{2}, \frac{\sqrt{3}}{2}) \;/\; x_B^2 + y_B^2 = 1 \rightarrow x_B^2 = 1 - \frac{1}{4} = \frac{3}{4} \rightarrow x_B = \pm\frac{\sqrt{3}}{2}$

$B(-\frac{\sqrt{3}}{2}, \frac{1}{2}) \qquad C(0, -1) \qquad S = \frac{1}{2}\left| x_A(y_B - y_C) + x_B(y_C - y_A) + x_C(y_A - y_B)\right|$

الف)

$S = \frac{1}{2}\left| \frac{1}{2}(\frac{1}{2} + 1) + -\frac{\sqrt{3}}{2}(-1 - \frac{\sqrt{3}}{2}) + 0(\frac{\sqrt{3}}{2} - \frac{1}{2})\right| = \frac{3 + \sqrt{3}}{4}$

ب)

$AB = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2} = \sqrt{(\frac{1}{2} + \frac{\sqrt{3}}{2})^2 + (\frac{\sqrt{3}}{2} - \frac{1}{2})^2} = \sqrt{2}$

$AC = \sqrt{(\frac{1}{2})^2 + (\frac{\sqrt{3}}{2} + 1)^2} = \sqrt{2 + \sqrt{3}} \qquad BC = \sqrt{\frac{3}{4} + \frac{9}{4}} = \sqrt{3} \qquad P = \sqrt{2} + \sqrt{3} + \sqrt{2 + \sqrt{3}}$

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④

$S = \frac{1}{2} \times AB \times BC \times \sin A \rightarrow \frac{1}{2} \times \log_3^{16} \times \log_2^{27} \times \underbrace{\sin 30}_{\frac{1}{2}} = \frac{1}{4} \times \log_3^{16} \times \log_2^{27}$

$\log_3^{16} = 4\log_3^2 \qquad \log_2^{27} = 3\log_2^3 \qquad \rightarrow S = 3\log_3^2 \times \log_2^3$

$S \rightarrow 3 \times \frac{1}{\cancel{\log_2^3}} \times \cancel{\log_2^3} = \boxed{3} \rightarrow$ SAB

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$f(x) = 2x^4 - x^2 - \sin^2\alpha(2\sin^2\alpha - 1)$

$(x - \cos\alpha) = 1$

$f(\cos\alpha) = 1 \rightarrow 2(\cos\alpha)^4 - (\cos\alpha)^2 - \sin^2\alpha(2\sin^2\alpha - 1)$

$2\cos^4\alpha - \cos^2\alpha - 2\sin^4\alpha + \sin^2\alpha = 1 \longrightarrow \sin^2\alpha = 1 - \cos^2\alpha$

$\rightarrow 2\cos^4\alpha - \cos^2\alpha - 2(1 - \cos^2\alpha)^2 + (1 - \cos^2\alpha) = 1$

$\rightarrow 2\cos^4\alpha - \cos^2\alpha - 2 + 4\cos^2\alpha - 2\cos^4\alpha + 1 - \cos^2\alpha = 1$

$(2\cos^4 - 2\cos^4) = 0 \longrightarrow 2\cos^2\alpha - 1 = 1 \rightarrow \cos^2\alpha = 1 \rightarrow \cos\alpha = \pm 1$

$\cos\alpha = 1 \longrightarrow \sin\alpha = 0$

$\cos\alpha = -1 \longrightarrow \sin\alpha = 0 \rightarrow \sin 2\alpha = 0 \rightarrow \sin 2\alpha = 0$

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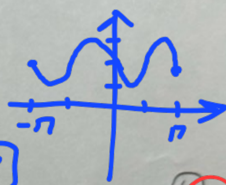

الف) $T = \pi$ $R_f = [1, 3]$

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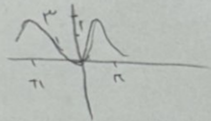
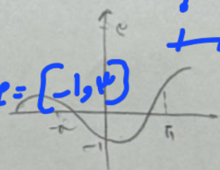
اما

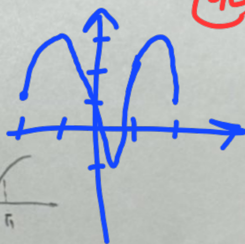
$y = -2\sin x\cos x + 2 \rightarrow x \in [-\pi, \pi] \rightarrow \sin(2x) = 2\sin x\cos x \rightarrow \frac{1}{2}\sin(2x)$

$\rightarrow y = -2 \times \frac{1}{2}\sin(2x) + 2 = -\sin(2x) + 2 \rightarrow$ دوره تناوب $\frac{2\pi}{2} = \pi$

ب) $T = \frac{2\pi}{1}$ شکل ب)







شکل ب) $T = 2$ $R_f = [-1, 3]$

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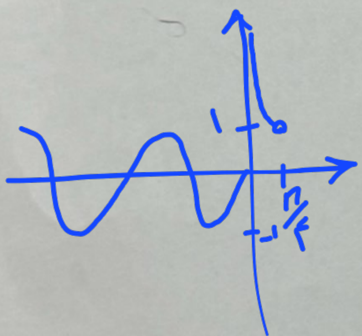

$R_f = [-1, +\infty)$

$f(x) = 4 \rightarrow x = \frac{\pi}{10}$

$f(x) = 0 \rightarrow x = \frac{\pi}{2}$

$\sin(cx)\cos(cx) = \frac{1}{2}\sin(2cx)$

$\rightarrow a + \frac{b}{2}\sin(2cx)\cos(2cx)$

$\rightarrow \sin(2cx)\cos(2cx) = \frac{1}{4}\sin(4cx)$

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$\rightarrow f(x) = a + \frac{b}{4}\sin(4cx) \longrightarrow A = \frac{b}{4}$ $f_{max} = 4$ $f_{min} = 0$ $a = \frac{4+0}{2} = 2$

$\frac{b}{4} = 2 \rightarrow b = 8$ $A = \frac{4-0}{2} = 2$

$\frac{\pi}{2} - \frac{\pi}{10} = \frac{4\pi}{10} = \frac{2\pi}{5} \rightarrow \frac{T}{2} = \frac{2\pi}{5} \rightarrow T = \frac{4\pi}{5} \rightarrow \frac{\pi}{2c} = \frac{4\pi}{5} \rightarrow c = \frac{5}{8} \rightarrow \frac{bc}{a} = \frac{8(\frac{5}{8})}{2} = \frac{5}{2}$

$C = \frac{5}{4}$

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(سارا امینی‌یار)

9

$a = \dfrac{2-(-4)}{2} = 3 \qquad c = \dfrac{2-4}{2} = -1$

$T = \dfrac{2\pi}{b} = \pi \longrightarrow b = 2$

$f(x) = 3\cos(2x) - 1 = 0 \longrightarrow \cos(2x) = \dfrac{1}{3} \longrightarrow x = \alpha \longrightarrow 2\alpha = \arccos\dfrac{1}{3}$

$|\tan(2\alpha)| = 2\sqrt{2}$ ✓ 2

$\hookrightarrow \dfrac{\sin(2\alpha)}{\cos(2\alpha)}$

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$f(-34.5) \longrightarrow f \quad (-1,-1) \longrightarrow (0,2) \longrightarrow (2,0)$

$f(-34.5) = f(-36 + 1.5) = f(1.5)$

$-34.5 + 3n \in [-1, 2]$

$-1 \leq -34.5 + 3n \leq 2 \longrightarrow 11.16 \leq n \leq 12.166 \longrightarrow n = 12$

$x' = -34.5 + 3(12) = 1.5 \longrightarrow f(-34.5) = f(1.5)$

$(0,2) \quad (2,0) \longrightarrow m = \dfrac{0-2}{2-0} = -1 \longrightarrow f(x) = 2 - x \longrightarrow 0 \leq x \leq 2$

$f(1.5) = 2 - 1.5 = 0.5$

$$f(x) = \begin{cases} -\frac{1}{2}x + 1 & 0 \leq x \leq 2 \\ 2x + 2 & -1 \leq x < 0 \end{cases}$$

$$\boxed{f(1.5) = \frac{1}{4}}$$