۱۹٫۵

«مبینا الحسنی» «دوازدهم ـ رشته A . مجید»

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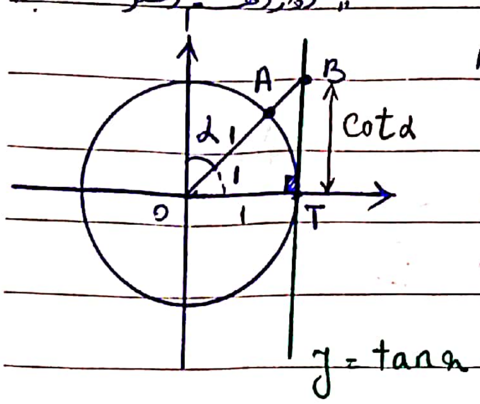

$AB = ?$

$1 + \alpha = \frac{\pi}{۲} \longrightarrow 1 = \frac{\pi}{۲} - \alpha$

$\tan 1 = \tan(\frac{\pi}{۲} - \alpha) = \cot\alpha = BT$

$OT = OA =$ (شعاع دایره مثلثاتی) $R = 1$

$OB^۲ = BT^۲ + OT^۲$

$OB^۲ = \cot^۲\alpha + 1 = \frac{1}{\sin^۲\alpha} \Rightarrow OB = \frac{1}{\sin\alpha}$

$OB = \overset{1}{\cancel{OA}} + AB \Rightarrow AB = \frac{1}{\sin\alpha} - 1 = \frac{1 - \sin\alpha}{\sin\alpha}$ ✓

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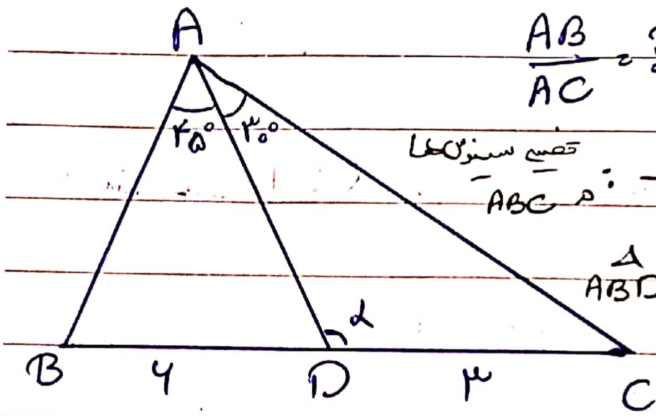


$\frac{AB}{AC} = ?$

قضیه سینوس‌ها در $\overset{\triangle}{ABC}$: $\frac{AB}{\sin\hat{C}} = \frac{AC}{\sin\hat{B}} \Rightarrow \frac{AB}{AC} = \frac{\sin\hat{C}}{\sin\hat{B}}$

۲

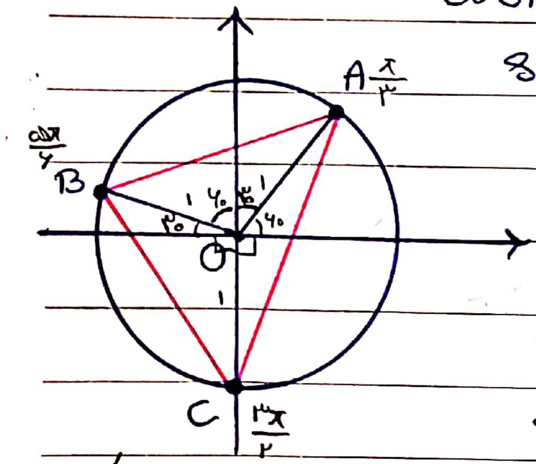
$\overset{\triangle}{ABD}$ در: $\frac{۶}{\frac{\sqrt{۲}}{۲}} = \frac{AD}{\sin B} \Rightarrow \frac{AD}{\sin B} = ۶\sqrt{۲}$

$\overset{\triangle}{ADC}$ در: $\frac{۳}{\frac{1}{۲}} = \frac{AD}{\sin C} \Rightarrow \frac{AD}{\sin C} = ۶$

$\Rightarrow$

$\frac{\sin\hat{C}}{\sin\hat{B}} = \frac{۶\sqrt{۲}}{۶} = \sqrt{۲} \Rightarrow \frac{AB}{AC} = \frac{\sin\hat{C}}{\sin\hat{B}} = \sqrt{۲}$ ✓

۳ـ



$\cos A = \frac{1}{۲} \longrightarrow A = \frac{\pi}{۳}$

$\sin B = \frac{1}{۲} \longrightarrow B = \frac{۵\pi}{۶}$

الف) $? = S_{\overset{\triangle}{ABC}}$

$S_{\overset{\triangle}{ABC}} = S_{\overset{\triangle}{AOC}} + S_{\overset{\triangle}{BOC}} + S_{\overset{\triangle}{BOA}}$

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$S_{\overset{\triangle}{AOC}} = \frac{1}{۲} \times 1 \times 1 \times \sin ۱۵۰° = \frac{1}{۴}$

$S_{\overset{\triangle}{BOC}} = \frac{1}{۲} \times 1 \times 1 \times \sin ۱۲۰° = \frac{\sqrt{۳}}{۴}$

$S_{\overset{\triangle}{BOA}} = \frac{1}{۲} \times 1 \times 1 \times \sin ۹۰ = \frac{1}{۲}$

$\Rightarrow \boxed{S_{\overset{\triangle}{ABC}} = \frac{۳ + \sqrt{۳}}{۴}}$ ✓

ب) $P_{\overset{\triangle}{ABC}}$

قضیه کسینوس‌ها: $AC^۲ = 1 + 1 - ۲\cos ۱۵۰ = ۲ + ۲ \times \frac{\sqrt{۳}}{۲} = ۲ + \sqrt{۳}$

$\boxed{AC = \sqrt{۲ + \sqrt{۳}}}$

$BC^۲ = 1 + 1 - ۲\cos ۱۲۰ = ۲ + ۲ \times \frac{1}{۲} = ۳$

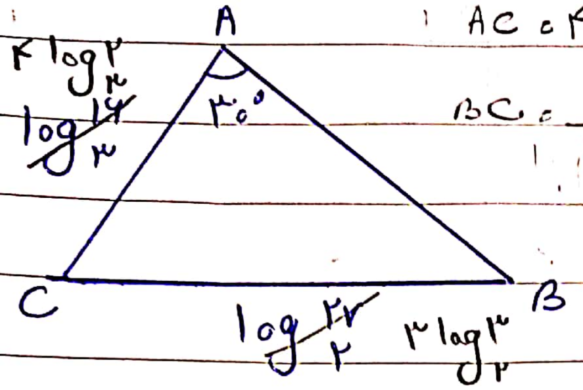
$\boxed{BC = \sqrt{۳}}$

$AB^۲ = 1 + 1 - ۲\cos ۹۰ = ۲$

$\boxed{AB = \sqrt{۲}}$

$\Rightarrow \boxed{P_{ABC} = \sqrt{۲} + \sqrt{۳} + \sqrt{۲ + \sqrt{۳}}}$ ✓

-۴



$AC = ۴\log_{۳} ۲$

$BC = \dfrac{۳}{\log_{۳} ۲}$

۲ ✓

$S_{\hat{ABC}} = \dfrac{۱}{۲} AB \cdot AC \cdot \overset{\sin ۳۰°}{\sin\alpha} =$

$۴\cancel{\log_{۳} ۲} \times \dfrac{۳}{\cancel{\log_{۳} ۲}} \times \dfrac{۱}{۲} \times \dfrac{۱}{۲} = \boxed{۳}$

-۵

$x = \cos\alpha$

$۲x^{۴} - x^{۲} - \sin^{۲}(\alpha)(۲\sin^{۲}\alpha - ۱) \rightarrow x = \cos\alpha$

$۲x^{۴} - x^{۲} + \sin^{۲}(\alpha)\cos ۲\alpha = ۱$ $\quad R = ۱ \rightarrow \sin ۲\alpha = ?$

$۲\cos^{۴}\alpha - \cos^{۲}\alpha + \sin^{۲}(\alpha)\cos ۲\alpha = ۱$

$\cos^{۲}\alpha(۲\cos^{۲}\alpha - ۱) + \sin^{۲}(\alpha)\cos ۲\alpha = ۱$

$\cos^{۲}\alpha\cos ۲\alpha + \sin^{۲}\alpha\cos ۲\alpha = \cos ۲\alpha(\cos^{۲}\alpha + \sin^{۲}\alpha) = ۱$

$\Rightarrow \cos ۲\alpha = ۱ \Rightarrow \sin^{۲} ۲\alpha + \underbrace{\cos^{۲} ۲\alpha}_{۱} = ۱ \Rightarrow \boxed{\sin ۲\alpha = 0}$

۲ ✓

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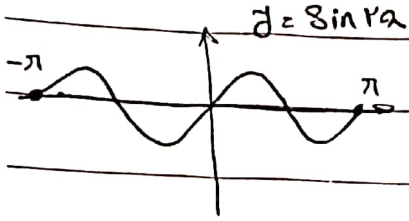
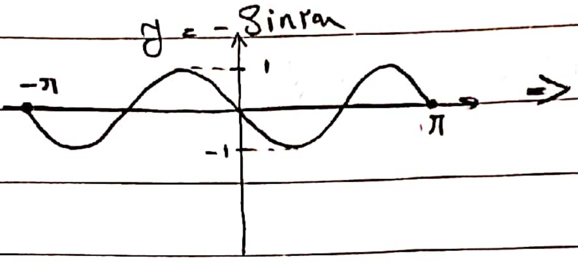
الف) $y = -2\sin x\cos x + 2$ $[-\pi, \pi]$

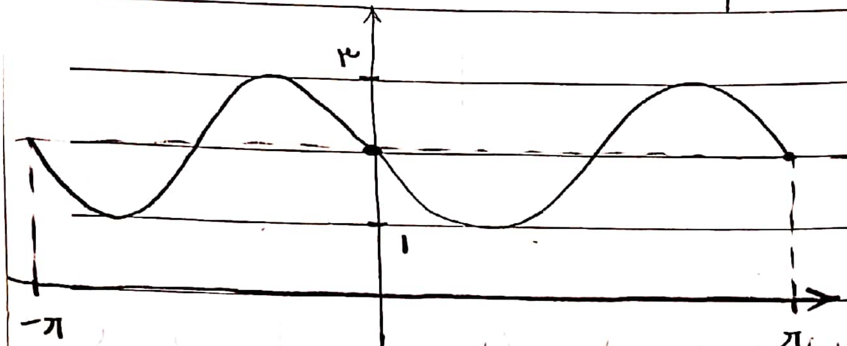
$y = -\sin 2x + 2$ $T = \frac{2\pi}{2} = \pi$ ✓

$y = \sin 2x$



$y = -\sin 2x$





$y = -\sin 2x + 2$

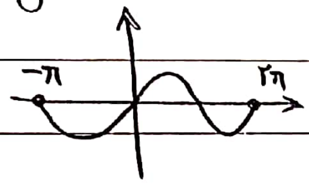
۱٫۵

✓

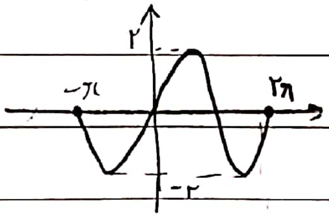
ب) $y = 2\cos(x + \frac{1}{2})\pi + 1$ $[-\pi, 2\pi]$

$y = 2\cos(\frac{\pi}{2} + x) + 1 \rightarrow y = -2\sin x + 1$ $T = \frac{2\pi}{\pi} = \boxed{2\pi}$

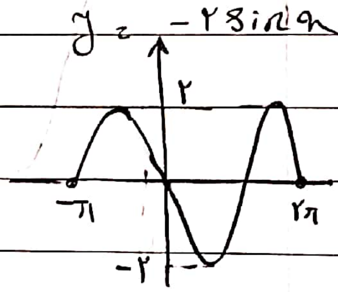
$y = \sin x$



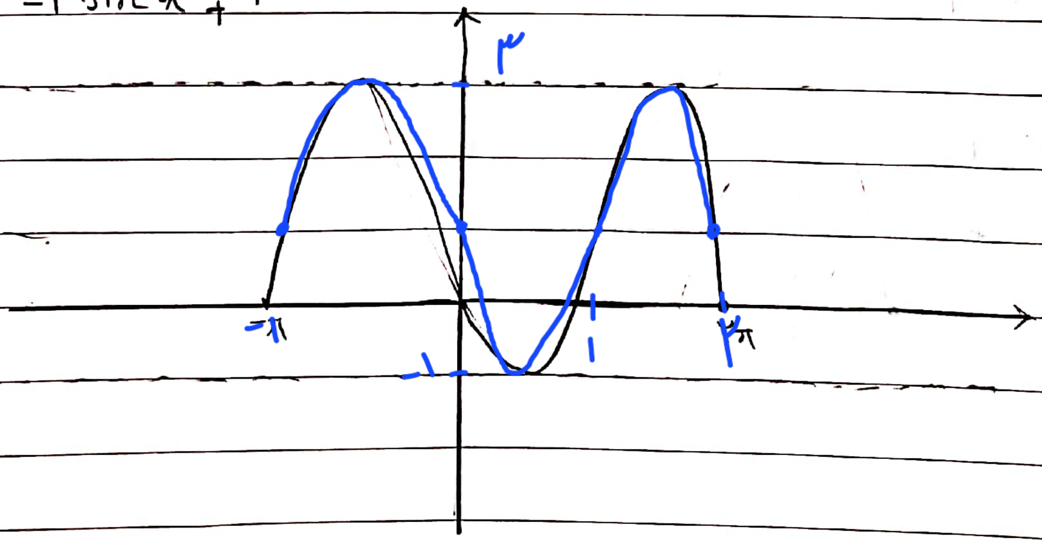
$y = 2\sin x$



$y = -2\sin x$



$y = -2\sin x + 1$



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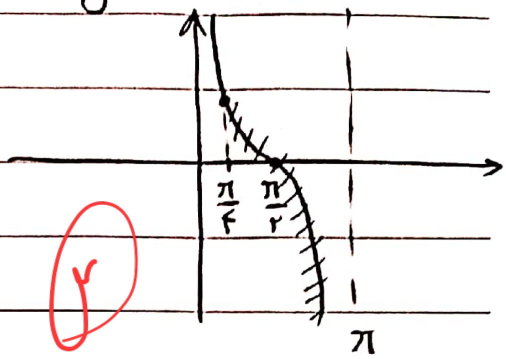
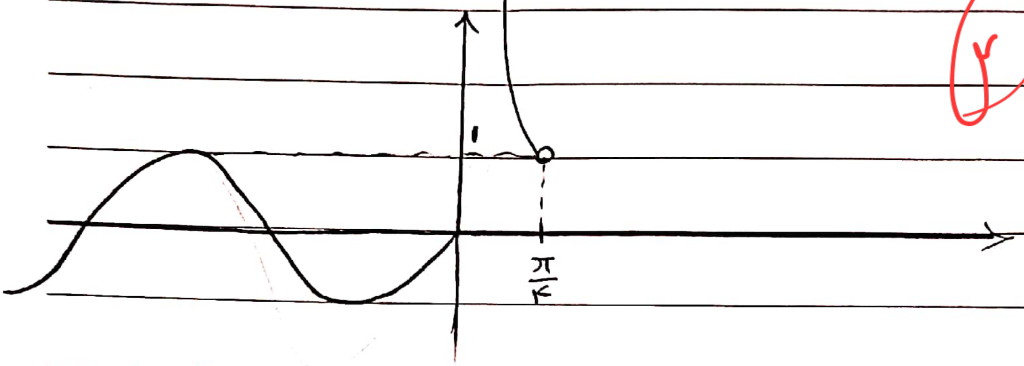
$f(x) = \begin{cases} \cot x & : 0 < x < \frac{\pi}{۴} \\ \sin x & : x \leq ۰ \end{cases}$

$* \; y = \cot x$





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10

برد تابع :

$R_f = [-1, +\infty)$

✓

$f(x) = a + b \underbrace{\sin(cx)\cos(cx)}_{\frac{1}{۲}\sin(۲cx)}\cos(۲cx)$

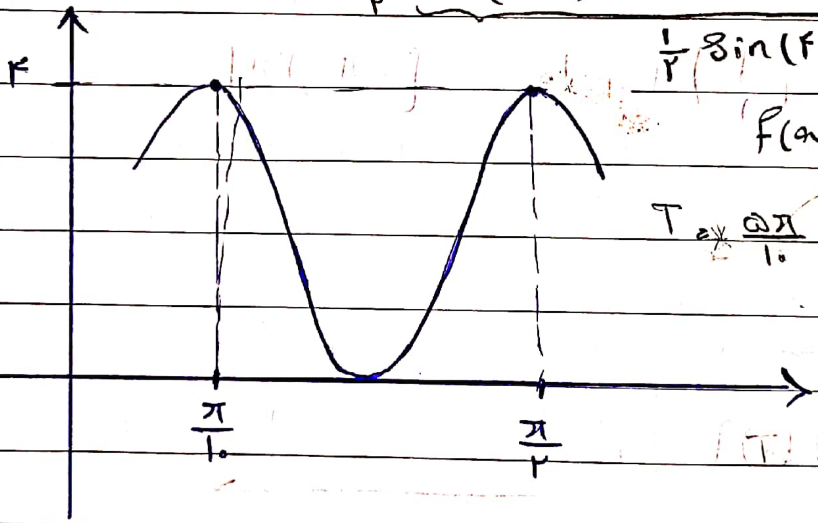
$\frac{1}{۲}\sin(۴cx)$

$\frac{bc}{a} = ?$ _۸

$(c > ۰)$

$(b < ۰)$

$f(x) = a + \frac{1}{۴} b \sin(۴cx)$



$T = \frac{۵\pi}{۱۰} - \frac{\pi}{۱۰} = \frac{۴\pi}{۱۰} = \frac{۲\pi}{۵} = \frac{۲\pi}{۴c}$

$\Rightarrow \boxed{c = \frac{۵}{۴}}$

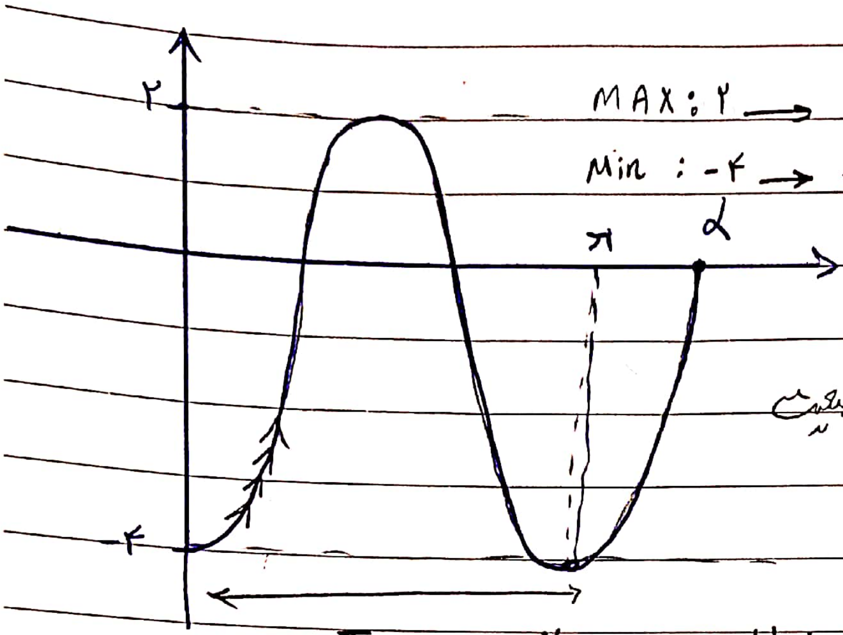
۲

$\text{Min}: a - \left|\frac{b}{۴}\right| = ۰ \Rightarrow a = \left|\frac{b}{۴}\right|$

$\text{Max}: a + \left|\frac{b}{۴}\right| = ۴ \Rightarrow ۲a = ۴ \Rightarrow a = ۲ , \frac{b}{۴} = ۲ \Rightarrow b = ۸$

$\frac{bc}{a} = \frac{۸ \times \frac{۵}{۴}}{۲} = \boxed{۵}$ ✓

$f(x) = a\cos bx + \overset{-۱}{c} \longrightarrow |\tan ۲\alpha| = ?$ ۹-



$MAX: ۲ \longrightarrow |a| + c = ۲$; $۲c = -۲$

$Min: -۴ \longrightarrow -|a| + c = -۴$ | $\boxed{c = -۱}$

$|a| = ۳$

cos پس از صفر بالا رفته پس وضعیت

غیرعادی است پس $ab < ۰$

$T = \pi \longrightarrow \frac{۲\pi}{|b|} = \pi \Rightarrow |b| = ۲ \longrightarrow b = ۲$ cos منفی خورده است

$a = -۳ \Leftarrow$

$f(x) = -۳\cos ۲x - ۱ = ۰ \Rightarrow \cos ۲x = \frac{-۱}{۳}$

$\boxed{|\tan ۲x| = \sqrt{۸}}$ ✓

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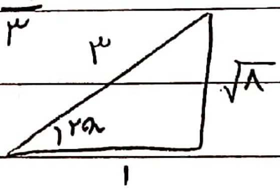
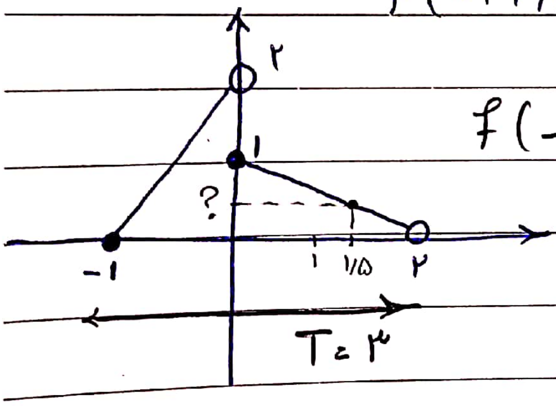


$f(-۳۴/۵) = ?$ ۱۰-



$f(-۳۴/۵) = f(-۱/۵) = f(۱/۵)$

۲

$m = \frac{-۱}{۲} \Rightarrow y = \frac{-۱}{۲}(x - ۲)$

$y = \frac{-۱}{۲}x + ۱$

$f(\frac{۳}{۲}) = \frac{-۱}{۲} \times \frac{۳}{۲} + ۱ = \frac{-۳}{۴} + ۱ = \frac{۱}{۴}$ ✓