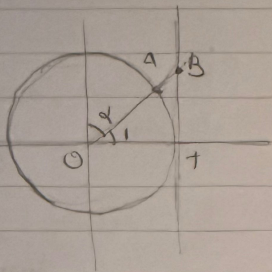
Date No

۱۹٫۲۵

آیلا میرودینا

۱)



A | $\cos(\frac{\pi}{2}-\alpha) \to \sin\alpha$

$\sin(\frac{\pi}{2}-\alpha) \to \cos\alpha$

B | 1

$\tan(\frac{\pi}{2}-\alpha) \to \cot\alpha$

$$AB = \sqrt{(\sin\alpha - 1)^2 + (\cos\alpha - \cot\alpha)^2}$$

$$AB = \frac{\sin x - 1}{\sin x}$$

۱٫۷۵

$\sin^2\alpha + 1 - 2\sin\alpha$

$$\cos\alpha - \cot\alpha = \cos\alpha - \frac{\cos\alpha}{\sin\alpha} = \frac{\cos\alpha(\sin\alpha - 1)}{\sin\alpha}$$

توان ۲ $\frac{\cos^2\alpha(\sin^2\alpha + 1 - 2\sin\alpha)}{\sin^2\alpha}$

$$= \frac{\sin^2\alpha(\sin^2\alpha + 1 - 2\sin\alpha) + (1 - \sin^2\alpha)(\sin^2\alpha + 1 - 2\sin\alpha)}{\sin^2\alpha} = \frac{1 + \sin^2\alpha - 2\sin\alpha}{\sin^2\alpha}$$

$$AB = \sqrt{\frac{(\sin x - 1)^2}{\sin^2}} \quad AB = \frac{\sin x - 1}{\sin x} \to \frac{1 - \sin\alpha}{\sin\alpha}$$

آیلا معز و ثنیا

۲.

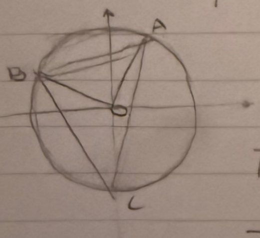
$$\frac{\frac{\sqrt{3}}{2}}{6} = \frac{\sin(180-\alpha)}{AB} \rightarrow \frac{\sqrt{3}}{12} = \frac{\sin\alpha}{AB} \rightarrow AB = \frac{12\sin\alpha}{\sqrt{3}}$$

$$\frac{\frac{1}{2}}{2} = \frac{\sin\alpha}{AC} \rightarrow \frac{1}{4} = \frac{\sin\alpha}{AC} \rightarrow AC = 4\sin\alpha$$

$$\rightarrow \frac{AB}{AC} = \frac{\frac{12\sin\alpha}{\sqrt{3}}}{\frac{4\sin\alpha}{1}} = \frac{3}{\sqrt{3}} = \sqrt{3}$$ ✓

۲

۳. ب



$$A\left|\begin{matrix}\frac{1}{2}\\ \frac{\sqrt{3}}{2}\end{matrix}\right. \quad B\left|\begin{matrix}-\frac{\sqrt{3}}{2}\\ \frac{1}{2}\end{matrix}\right. \quad C\left|\begin{matrix}0\\ -1\end{matrix}\right.$$

$$\overline{AB} = \sqrt{\left(\frac{1+\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}-1}{2}\right)^2} = \sqrt{2}$$

$$\overline{BC} = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(-\frac{3}{2}\right)^2} = \sqrt{3}$$

$$\overline{AC} = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}+2}{2}\right)^2} = \sqrt{2+\sqrt{3}}$$

۲

$$\sqrt{2} + \sqrt{3} + \sqrt{2+\sqrt{3}}$$ ✓

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س. الف)

$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$\to \cos B = \frac{2 + 3 - 2 - \sqrt{3}}{2 \times \sqrt{3} \times \sqrt{2}} \to \frac{3 - \sqrt{3}}{2\sqrt{6}} , \frac{\sqrt{3}(\sqrt{3}-1)}{\sqrt{3} \times 2\sqrt{2}} , \cos B = \frac{\sqrt{3}-1}{2\sqrt{2}}$

$\sin B = \frac{1 + \sqrt{3}}{2\sqrt{2}}$

$S = \frac{1}{2} \times \sqrt{3} \times \sqrt{2} \times \frac{1+\sqrt{3}}{2\sqrt{2}} = \frac{3 + \sqrt{3}}{4}$ ✓

۴.

$a^2 = b^2 + c^2 - 2bc\cos(30) \to c^2 - \sqrt{3}\,bc + b^2 - a^2 = 0$

(۲)

$c = \frac{\sqrt{3}b \pm \sqrt{4a^2 - b^2}}{2}$ $S = \frac{1}{2} \times \sin A \times bc = \frac{b^2\sqrt{3} \pm b\sqrt{4a^2 - b^2}}{}$

$\hat{S} = \frac{1}{2} \times AB \times AC \times \sin 30$

$\to S = 2\sqrt{3}(\log_3 2)^2 + \sqrt{9 - 4(\log_3 2)^2}$

$S = \frac{1}{2} \times \log_2 3^3 \times \log_3 2^4 \times \frac{1}{2} = \frac{1}{4} \times 4 \times 3 \times \log_2 3 \times \log_3 2 = \boxed{3}$

۵.

$x = \cos\alpha$ $2\cos^4\alpha - \cos^2\alpha - \sin^2\alpha(2\sin^2\alpha - 1) = 1$

$\cos^2\alpha(2\cos^2\alpha - 1) - \sin^2\alpha(2\sin^2\alpha - 1) = 1 \to \cos 2\alpha(\cos^2\alpha + \sin^2\alpha) = 1$

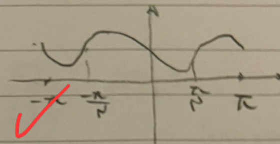
$\cos 2\alpha$ $-\cos 2\alpha$

$\cos 2\alpha = 1 \to \sin 2\alpha = 0$ ✓

(۳)

آیلا میرزدنیا

4. الف) $T = \frac{2\pi}{|b|} = \frac{2\pi}{2} = \pi$ ✓ $y = -2\sin x\cos x + 2 \rightarrow y = -\sin 2x + 2$



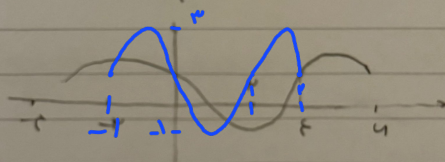
$R_f = [1, 3]$

1,5

ب) $y = 2\cos\pi(x + \frac{1}{2}) + 1 = 2\cos(\pi x + \frac{\pi}{2}) + 1 = -2\sin\pi x + 1$

$R_f = [-1, 3]$

$T = \frac{2\pi}{|b|} = \frac{2\pi}{\pi} = 2$ ✓



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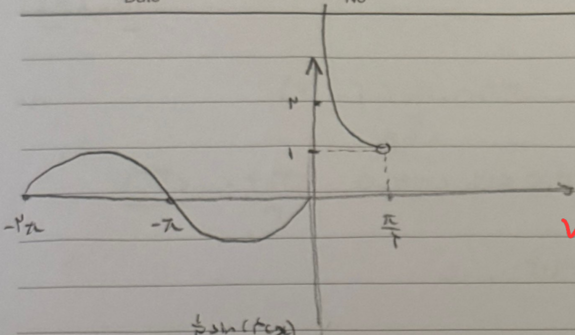
آیلا فیروزنیا



۷. $R_f = [-1, +\infty)$ ✓ (۲)

۸. $f(x) = b\sin(cx)\cos(cx)\cos(2cx) + a$

$\rightarrow f(x) = \frac{b}{4}\sin(4cx) + a \quad \begin{cases} \left|\frac{b}{4}\right| + a = 4 \\ -\left|\frac{b}{4}\right| + a = 0 \end{cases}$ (۲)

$2a = 4 \rightarrow a = 2, \quad |b| = 8 \rightarrow b = 8$

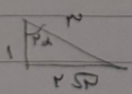
$T = \frac{\pi}{2} - \frac{\pi}{4} = \frac{2\pi}{5} = \frac{2\pi}{|4c|} \rightarrow |c| = \frac{5}{4} \xrightarrow{c>0} \frac{5}{4} = c$

$\frac{bc}{a} = \frac{8 \times \frac{5}{4}}{2} = 5$ ✓

۹. چون برد تابع صفر بود ۳ شدن و بعد یه واحد منتقل شده به پایین) $a = -3$
و $c = -1$ نسبت به محور قرینه

$\frac{2\pi}{|b|} = \frac{2\pi}{\pi} = 2 \rightarrow f(x) = -3\cos(2x) - 1 \rightarrow f(\alpha) = -3\cos 2\alpha - 1 = 0$ (۲)

$\cos 2\alpha = -\frac{1}{3}$



$\rightarrow \tan(2\alpha) = \frac{2\sqrt{2}}{1} = \sqrt{8}$ ✓

آیلا فروتنیا

۱۰. $T=۳ \rightarrow f(x) = f(x+۳) \rightarrow -۳۴{,}۵ + ۳n \in [-۱, ۲)$

$n \in \mathbb{Z}$

$-۱ \leq -۳۴{,}۵ + ۳n < ۲ \rightarrow +۳۳{,}۵ \leq ۳n < ۳۶{,}۵ \rightarrow \frac{۳۳{,}۵}{۳} \leq n < \frac{۳۶{,}۵}{۳}$

$\rightarrow n = ۱۲ \rightarrow x' = -۳۴{,}۵ + ۳(۱۲) = ۱{,}۵$

$f(۱{,}۵) = f(-۳۴{,}۵) \rightarrow$ طبق شکل $\rightarrow -\frac{۱}{۲}x + ۱ \rightarrow -\frac{۱}{۲} \times \frac{۳}{۲} + \frac{۲}{۲} =$

$= \frac{-۳}{۴} + \frac{۴}{۴} = \frac{۱}{۴}$