عربی

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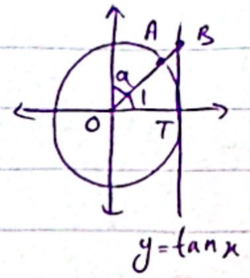

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AB ?

$BT = \tan \hat{O}_1 \xrightarrow{\hat{O}_1 + \alpha = 90} BT = \cot \alpha$

$\overset{\Delta}{OBT} : \hat{T} = 90 \rightarrow OT^2 + BT^2 = OB^2$

$\xrightarrow{OT=1} 1 + \cot^2\alpha = OB^2 \rightarrow$

$OB^2 = \frac{1}{\sin^2\alpha} \rightarrow OB = \frac{1}{\sin\alpha}$

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$OB = OA + AB \xrightarrow{OA=1}$

$\frac{1}{\sin\alpha} = 1 + AB \rightarrow AB = \frac{1 - \sin\alpha}{\sin\alpha}$

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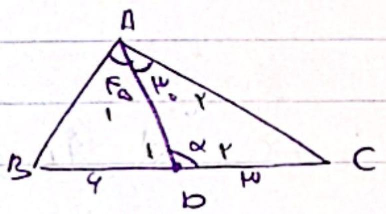

$\frac{AB}{AC} = ?$

$\overset{\Delta}{ABD}$ قضیهٔ سینوس ها در : $\frac{BD}{\sin \hat{A}_1} = \frac{AB}{\sin \hat{D}_1}$

$\xrightarrow[\sin(\hat{D}_1) = \sin(180 - \hat{D}_2) = \sin \hat{D}_2]{\hat{D}_1 + \hat{D}_2 = 180} \frac{6}{\frac{\sqrt{2}}{2}} = \frac{AB}{\sin\hat{D}_2} \rightarrow AB = 6\sqrt{2} \sin \hat{D}_2$

$\overset{\Delta}{ADC}$ قضیهٔ سینوس در : $\frac{DC}{\sin \hat{A}_2} = \frac{AC}{\sin \hat{D}_2} \rightarrow AC = 6 \sin \hat{D}_2$

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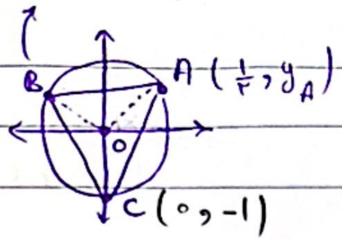
جواب : $\frac{AB}{AC} = \frac{6\sqrt{2} \cancel{\sin \hat{D}_2}}{6 \cancel{\sin \hat{D}_2}} = \sqrt{2}$

Subject $(x_B, \frac{1}{2})$



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$A(\frac{1}{2}, y_A) \rightarrow \cos\hat{A} = \frac{1}{2}$ (۳

الف)

$\xrightarrow{\text{ربع اول } A} y_A = \sin A = \frac{\sqrt{3}}{2}$

$B(x_B, \frac{1}{2}) \rightarrow \sin\hat{B} = \frac{1}{2} \xrightarrow{\text{ربع دوم } B} x_B = \cos B = \frac{-\sqrt{3}}{2}$

$$S_\Delta = \frac{1}{2}\begin{vmatrix} x_A & y_A & 1 \\ x_B & y_B & 1 \\ x_C & y_C & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 1 \\ 0 & -1 & 1 \end{vmatrix} = \frac{\sqrt{3}+3}{4}$$

۲

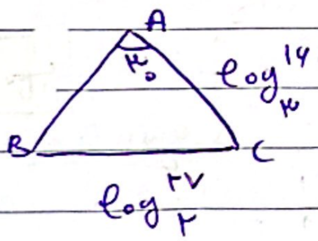
ب)

$$AB = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2} = \sqrt{2}$$

$$AC = \sqrt{(x_A - x_C)^2 + (y_A - y_C)^2} = \sqrt{2 + \sqrt{3}}$$

محیط $= \sqrt{3} + \sqrt{2} + \sqrt{2 + \sqrt{3}}$

$$BC = \sqrt{(x_B - x_C)^2 + (y_B - y_C)^2} = \sqrt{3}$$



$S_\Delta = ?$

قضیه سینوس ها: $\frac{BC}{\sin A} = \frac{AC}{\sin B} \rightarrow \frac{\log_2^{27}}{\frac{1}{2}} = \frac{\log_3^{16}}{\sin B}$ (۴

$\sin B = \frac{4\log_3^2}{2 \times 3\log_2^3}$

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$$S = \frac{1}{2} \times AB \times AC \times \sin 30° = \frac{1}{2} \times \log_2^{27} \times \log_3^{16} \times \frac{1}{2}$$

$$= \frac{1}{4} \times 3 \times 4 \times \log_2^3 \times \log_3^2 = \boxed{3}$$

s.a.m

$x - \cos\alpha = ۰ \rightarrow x = \cos\alpha \longrightarrow$ ۵_

$۲\cos^۴\alpha - \cos^۲\alpha - ۲\sin^۴\alpha + \sin^۲\alpha = ۱$

$\rightarrow ۲(\cos^۴\alpha - \sin^۴\alpha) + -۱(\cos^۲\alpha - \sin^۲\alpha) = ۱$

$۲(\cos^۲\alpha - \sin^۲\alpha)\underbrace{(\cos^۲\alpha + \sin^۲\alpha)}_{۱} + -(\cos^۲\alpha - \sin^۲\alpha) = ۱$

$۲\cos^۲\alpha - ۲\sin^۲\alpha - \cos^۲\alpha + \sin^۲\alpha = ۱$ ۲

$\cos^۲\alpha - \sin^۲\alpha = ۱$

$\cos ۲\alpha = ۱ \rightarrow \sin ۲\alpha = ۰$

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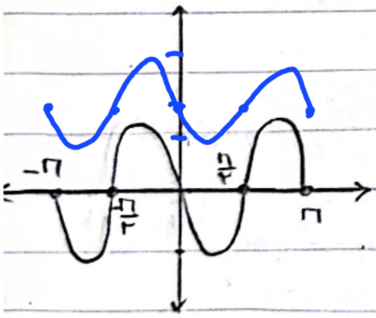
الف)

$[-\pi, \pi]$

$y = -۲\sin x\cos x + ۲ = ۲ - \sin ۲x$ $T = \frac{۲\pi}{|a|} = \frac{۲\pi}{۲} = \pi$



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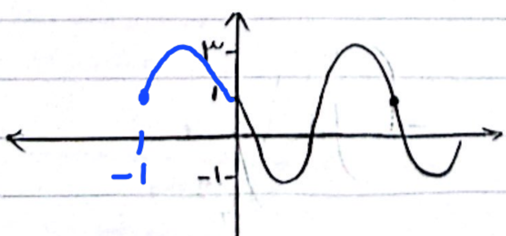
ب)

$[-\pi, ۲\pi]$

$y = ۲\cos(x + \frac{۱}{۴})\pi + ۱ = ۲\cos(\frac{\pi}{۴} + \pi x) + ۱ = -۲\sin \pi x + ۱$

$T = \frac{۲\pi}{\pi} = ۲$



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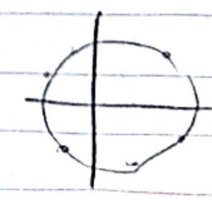

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$$f(x) = \begin{cases} \cot x & 0 < x < \frac{\pi}{4} \\ \sin x & x \leq 0 \end{cases}$$ ۷

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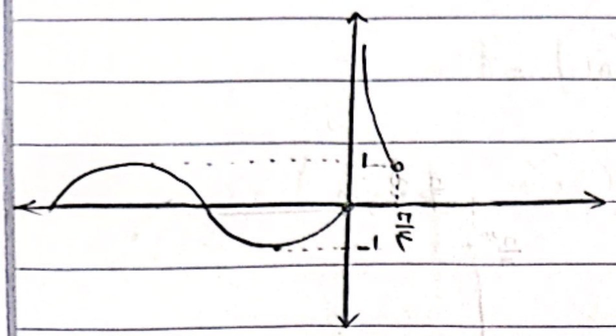

$\rightarrow \quad \Rightarrow R_{f(x)} = [-1, +\infty)$

$$f(x) = a + b\,\frac{\overset{\times 4}{\sin(cx)\cos(cx)}\cos(2cx)}{4} =$$ ۸

$$a + \frac{2b\sin(2cx)\cos(2cx)}{4} \rightarrow a + \frac{b\sin 4x}{4}$$

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$$\underbrace{\sin cx \cos cx}_{\frac{1}{2}\sin 2cx} \cos 2cx = \frac{1}{4}\sin 4cx \rightarrow a + \frac{b}{4}\sin 4cx$$

$$T = \frac{2\pi}{|a|} \rightarrow \frac{2\pi}{|4c|} = \frac{2\pi}{\omega} \rightarrow |4c| = \omega \xrightarrow{c>0} c = \frac{\omega}{4}$$

$$\begin{cases} a + \frac{b}{4} = 4 \\ a - \frac{b}{4} = 0 \end{cases} \rightarrow \begin{matrix} a = 2 \\ b = 8 \end{matrix} \rightarrow \boxed{\frac{bc}{a} = \omega}$$

۰/۵ ۰ ۸

s.a.m