نمونه سوال

۱- $\overline{OT} = OA = 1$, $\tan 1 = \cot\alpha$, $OB^2 = OT^2 + TB^2 \Rightarrow OB = \sqrt{\frac{\cos^2\alpha}{\sin^2\alpha}+1} = \sqrt{\frac{\sin^2\alpha+\cos^2\alpha}{\sin^2\alpha}} = \frac{1}{\sin\alpha} \Rightarrow AB = \frac{1}{\sin\alpha} - 1 = \frac{1-\sin\alpha}{\sin\alpha}$

۲- $\frac{6}{\sin 45} = \frac{\sqrt{2}\times 6\times 2}{2} = 6\sqrt{2}$, $\sin 45 = \frac{\sqrt{2}}{2}$, $\frac{AB}{\sin D} = 6\sqrt{2}$, $\frac{3}{\sin 30^\circ} = 6$, $\sin 30 = \frac{1}{2}$, $\frac{AC}{\sin\alpha} = 6$, $\sin\alpha = \sin D$

$\Rightarrow AB = 6\sqrt{2}\sin\alpha$, $AC = 6\sin\alpha$, $\frac{AB}{AC} = \frac{6\sqrt{2}\sin\alpha}{6\sin\alpha} = \sqrt{2}$

۳- $\sin B = \frac{1}{2} \Rightarrow B = 150^\circ$, $\cos A = \frac{1}{2} \Rightarrow A = 60^\circ$

$A\hat{O}B = 150^\circ - 60 = 90^\circ \Rightarrow O\hat{A}B = A\hat{B}O = 45^\circ$

$AB = \sqrt{2}$

$B\hat{O}C = 120^\circ = 90+30$

$BC = \sqrt{1^2+1^2-2\cos 120^\circ} = \sqrt{3}$, $\cos 120 = -\frac{1}{2}$

$A\hat{O}C = 150^\circ = 90+60$

$AC = \sqrt{1^2+1^2-2\cos 150^\circ} = \sqrt{2+\sqrt{3}}$, $\cos 150 = -\frac{\sqrt{3}}{2}$

الف) $A\hat{B}O = 45$

$O\hat{B}C = \frac{180-120^\circ}{2} = 30$

$\Rightarrow S_{ABC} = \sqrt{2}\times\frac{1}{2}\times\sqrt{3}\times\sin(\frac{90}{2}+30)$

$= \frac{1}{2}\times\sqrt{2}\times\sqrt{3}\times\frac{\sqrt{3}}{2} = \frac{3\sqrt{2}}{4}$, $\cos 30$

ب) $P_{ABC} = \sqrt{2+\sqrt{3}} + \sqrt{3} + \sqrt{2}$

۴-

$a^۲ = b^۲ + c^۲ - ۲bc\cos\alpha$

$c^۲ - ۲bc\cos\alpha + b^۲ - a^۲ = ۰$

$c^۲ - ۲(b\cos\alpha)c + (b^۲ - a^۲) = ۰$

$\Rightarrow c = \dfrac{۲b\cos\alpha \pm \sqrt{۴b^۲\cos^۲\alpha - ۴b^۲ + ۴a^۲}}{۲} \Rightarrow c = b\cos\alpha \pm \sqrt{a^۲ - b^۲\sin^۲\alpha}$

$S = \frac{۱}{۲} bc\sin\alpha = \frac{۱}{۲}\log_{۳^۲}^{۲۷} a\sqrt{۳}/۲ + \sqrt{(\log_۳ ۱۹)^۲ - ۹a\,۳/۴} = \sqrt{۳}/۱۲ + \sqrt{\log_۳^{۳۲} - ۲۷/۴}$

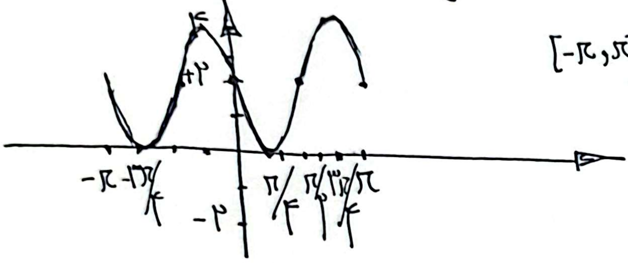
$$x^2(2x^2-1)-(1-\cos^2\alpha)(2-2\cos^2\alpha-1) \Rightarrow x^2(2x^2-1)-(1-\cos^2\alpha)(1-2\cos^2\alpha)$$

$$\cos^2\alpha(2\cos^2\alpha-1)-(1-\cos^2\alpha)(1-2\cos^2\alpha)=(2\cos^2\alpha-1)(\cancel{\cos^2\alpha}+1-\cancel{\cos^2\alpha}) \Rightarrow 2\cos^2\alpha-1=1$$ -۵

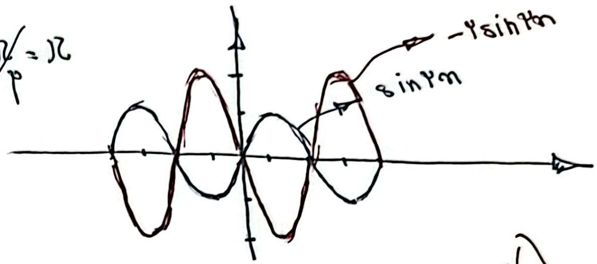
$2\cos^2\alpha=2 \Rightarrow \cos^2\alpha=1$ $\quad \sin^2\alpha=1-1=0$ $\quad \sin 2\alpha=2\sin\alpha\cos\alpha=0$

-۶

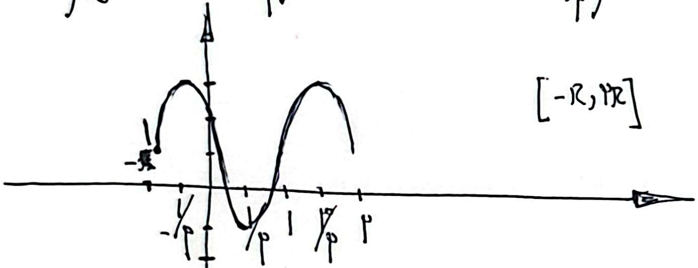
الف) $2\sin x\cos x=\sin 2x \Rightarrow y=-2\sin 2x+2$ $\quad T=\frac{2\pi}{|2|}=\frac{2\pi}{2}=\pi$



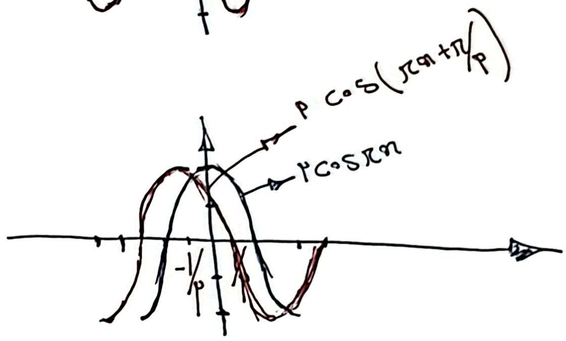
$[-\pi,\pi]$

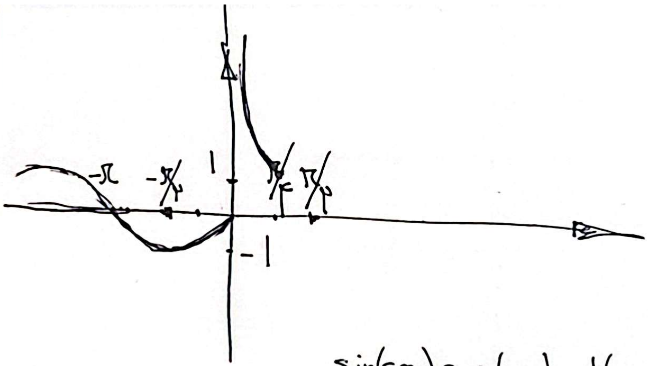


ب) $y=2\cos(x+\frac{1}{2})\pi+1=2\cos(\pi x+\frac{\pi}{2})+1$ $\quad T=\frac{2\pi}{\pi}=2$



$[-\pi,2\pi]$





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$\sin(cx)\cos(cx) = \frac{1}{2}\sin(2cx)$

$\frac{1}{2}\sin(2cx)\cos(2cx) = \frac{1}{4}\sin(4cx)$ $\Rightarrow f(x) = a + \frac{b}{4}\sin(4cx)$ -8

$\Rightarrow$

$x = \frac{\pi}{2} \rightarrow a + \frac{b}{4}\sin(2\pi c) = 4$

$x = \frac{\pi}{10} \rightarrow a + \frac{b}{4}\sin\left(\frac{2\pi}{5}c\right) = 4$

$\Rightarrow a + \frac{b}{4} = 4$ $\qquad \frac{2\pi}{5}c = \frac{\pi}{2} \Rightarrow c = \frac{5}{4}$

$a - \frac{b}{4} = 0 \Rightarrow a = \frac{b}{4}$ $\quad \begin{matrix} a = 2 \\ b = 8 \end{matrix} \Rightarrow = 5 = \frac{bc}{a}$

$T = \pi = \frac{2\pi}{|b|} \Rightarrow |b| = 2$ 9-

$c + |a| = 2$
$c - |a| = -4 \Rightarrow 2c = -2,\ c = -1 \Rightarrow |a| = 3 \Rightarrow a = -3$

$-3\cos 2\pi - 1 = -4$
$+1$

$f(x) = -3\cos 2x - 1$

$-3\cos 2x - 1 = 0 \qquad -3\cos 2\alpha = 1 \Rightarrow \cos 2\alpha = -\frac{1}{3}$

$\sin 2\alpha = -\sqrt{1 - \frac{1}{9}} = -\frac{\sqrt{8}}{3} \qquad |\tan 2\alpha| = \frac{\frac{2\sqrt{2}}{3}}{\frac{1}{3}} = 2\sqrt{2}$

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$-24.5 = -1 - 23.5$
$-23.5 = +2\times -11 + -0.5 \Rightarrow f(-1) = 0 \Rightarrow f(-1.5) = f(-23.5) = 1$