ترم ۲ فصل

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$OT = OA = 1$

$\tan 1 = \cot\alpha$

$OB^2 = OT^2 + TB^2 \Rightarrow OB = \sqrt{\frac{\cos^2\alpha}{\sin^2\alpha} + 1} = \sqrt{\frac{\sin^2\alpha + \cos^2\alpha}{\sin^2\alpha}} = \frac{1}{\sin\alpha} \Rightarrow AB = \frac{1}{\sin\alpha} - 1 = \frac{1 - \sin\alpha}{\sin\alpha}$

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$\frac{6}{\sin 45} = \frac{6}{\frac{\sqrt{2}}{2}} = \frac{\sqrt{2} \times 6 \times 2}{2} = 6\sqrt{2}$ $\quad \frac{AB}{\sin D} = 6\sqrt{2}$ $\quad \frac{3}{\sin 30^\circ} = 6$, $\sin 30^\circ = \frac{1}{2}$ $\quad \frac{AC}{\sin\alpha} = 6$ $\quad \sin\alpha = \sin D$

$\Rightarrow AB = 6\sqrt{2}\sin\alpha$ $\quad AC = 6\sin\alpha$ $\quad \frac{AB}{AC} = \frac{6\sqrt{2}\sin\alpha}{6\sin\alpha} = \sqrt{2}$

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$\sin B = \frac{1}{2} \Rightarrow B = 150^\circ$

$\cos A = \frac{1}{2} \Rightarrow A = 60^\circ$

$A\hat{O}B = 150^\circ - 60 = 90^\circ \Rightarrow O\hat{A}B = A\hat{B}O = 45^\circ$

$AB = \sqrt{2}$

$B\hat{O}C = 120^\circ = 90 + 30$

$BC = \sqrt{1^2 + 1^2 - 2\cos 120^\circ} = \sqrt{3}$, $\cos 120^\circ = -\frac{1}{2}$

$A\hat{O}C = 150^\circ = 90 + 60$

$AC = \sqrt{1^2 + 1^2 - 2\cos 150^\circ} = \sqrt{2 + \sqrt{3}}$, $\cos 150^\circ = -\frac{\sqrt{3}}{2}$

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$A\hat{B}O = 45$

$O\hat{B}C = \frac{180 - 120^\circ}{2} = 30$

$\Rightarrow S_{ABC} = \sqrt{2+\sqrt{3}} \times \frac{1}{2} \times \sqrt{2} \times \sin\left(\frac{\pi}{4} + 30\right)$

$= \frac{1}{2} \times \sqrt{2} \times \sqrt{3} \times \frac{\sqrt{3}}{2} =$ $\quad \frac{3\sqrt{2}}{4}$ $\cos 30$

ب)

$P_{ABC} = \sqrt{2 + \sqrt{3}} + \sqrt{3} + \sqrt{2}$

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$S = \frac{1}{2} + \frac{1}{4} + \frac{\sqrt{3}}{4} = \frac{3 + \sqrt{3}}{4}$

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$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$c^2 - 2bc\cos\alpha + b^2 - a^2 = 0$$

$$c^2 - 2(b\cos\alpha)c + (b^2 - a^2) = 0$$

$$\Rightarrow c = \frac{2b\cos\alpha \pm \sqrt{4b^2\cos^2\alpha - 4b^2 + 4a^2}}{2} \Rightarrow c = b\cos\alpha \pm \sqrt{a^2 - b^2\sin^2\alpha}$$

$$S = \frac{1}{2}bc\sin\alpha = \frac{1}{2}\log_3^{27} \times \sqrt{3}/2 + \sqrt{\left(\log_3^{9}\right)^2 - 9} \times 3/4 = \sqrt{3}/12 + \sqrt{\log_3^{27} - 3}/4$$

$$S_{\hat{ABC}} = \frac{1}{2} \times AB \times AC \times \sin 30^\circ = \frac{1}{2} \times \log_2^{16} \times \log_3^{9} \times \frac{1}{2} = \frac{1}{4}\log_2^{2^4} \times \log_3^{3^2}$$

$$S = \frac{1}{4} \times 4 \times 2 \times \log_2^{2} \times \log_3^{3} = 2$$

$\sin^2\alpha\left(4\cos^2\alpha-1\right)-\left(1-\cos^2\alpha\right)\left(4-4\cos^2\alpha-1\right) \Rightarrow \sin^2\alpha\left(4\cos^2\alpha-1\right)-\left(1-\cos^2\alpha\right)\left(1-4\cos^2\alpha\right)$

$\cos^2\alpha\left(4\cos^2\alpha-1\right)-\left(1-\cos^2\alpha\right)\left(1-4\cos^2\alpha\right)=\left(4\cos^2\alpha-1\right)\left(\cos^2\alpha+1-\cos^2\alpha\right) \Rightarrow 4\cos^2\alpha-1=1$

$4\cos^2\alpha=4 \Rightarrow \cos^2\alpha=1 \qquad \sin^2\alpha=1-1=0 \qquad \sin 2\alpha=2\sin\alpha\cos\alpha=0$

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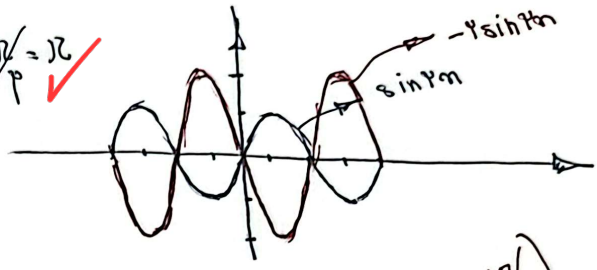
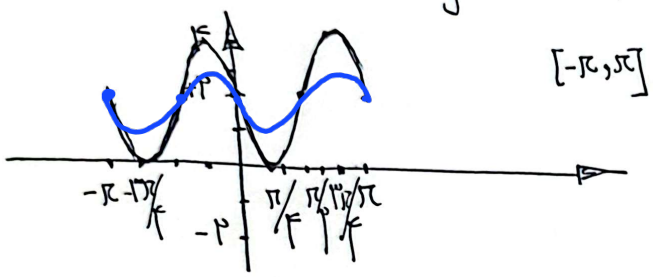
۶- الف) $2\sin\alpha\cos\alpha=\sin 2\alpha \Rightarrow y=-2\sin^2 x+2$

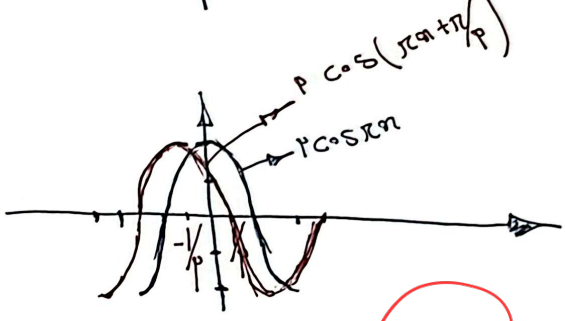
$T=\frac{2\pi}{|b|}=\frac{2\pi}{2}=\pi$

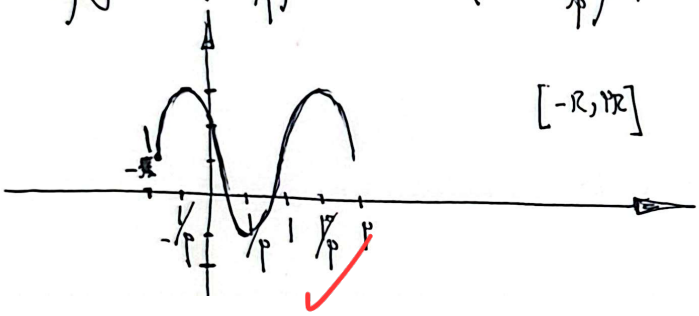




ب) $y=2\cos\left(x+\frac{1}{2}\right)\pi+1=2\cos\left(\pi x+\frac{\pi}{2}\right)+1$

$T=\frac{2\pi}{\pi}=2$





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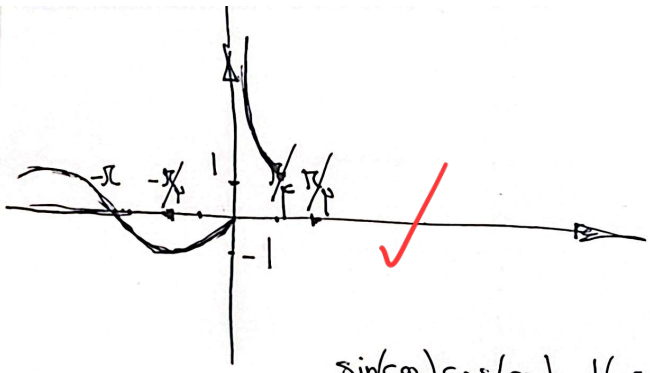

$R_f = [-1, +\infty)$ (۱,۷۵)

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$\sin(cx)\cos(cx) = \frac{1}{2}\sin(2cx)$

$\frac{1}{2}\sin(2cx)\cos(2cx) = \frac{1}{4}\sin(4cx)$

$\Rightarrow f(x) = a + \frac{b}{4}\sin(4cx)$

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$x = \frac{\pi}{2} \rightarrow a + \frac{b}{4}\sin(2\pi c) = 4$

$x = \frac{\pi}{10} \rightarrow a + \frac{b}{2}\sin(\frac{2\pi}{5}c) = 4$

$\Rightarrow a + \frac{b}{4} = 4$

$a - \frac{b}{4} = 0 \Rightarrow a = \frac{b}{4}$ $\quad a=2$ $\quad b=8$ $\Rightarrow$ ~~$\frac{4a}{2}$~~ $= 0 = \frac{bc}{a}$

$\frac{2\pi}{5}c = \frac{\pi}{2} \Rightarrow c = \frac{5}{4}$

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$T = \pi = \frac{2\pi}{|b|} \Rightarrow |b| = 2$

$c + |a| = 2$, $c - |a| = -4 \Rightarrow 2c = -2$, $c = -1 \Rightarrow |a| = 3 \Rightarrow a = -3$

$-3\cos 2\pi - 1 = -4$ ($\cos 2\pi \to +1$)

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$f(x) = -3\cos 2x - 1$

$-3\cos 2x - 1 = 0$ $\quad -3\cos 2\alpha = 1 \Rightarrow \cos 2\alpha = -\frac{1}{3}$

$\sin 2\alpha = -\sqrt{1 - \frac{1}{9}} = -\frac{\sqrt{8}}{3}$ $\quad |\tan 2\alpha| = \frac{2\sqrt{2}/3}{1/3} = 2\sqrt{2}$ ✓

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(5)

$-34.5 = -1 - 33.5$

$-33.5 = +3 \times -11 + -0.5 \Rightarrow f(-1) = 0 \Rightarrow f(-0.5) = f(-34.5) = 1$

$f(-34.5) = f(-36 + 1.5) = f(1.5)$

$$f(x) = \begin{cases} -\frac{1}{2}x + 1 & 0 \leq x \leq 2 \\ 2x + 2 & -1 \leq x < 0 \end{cases} \longrightarrow f(1.5) = -\frac{1}{2}\left(\frac{3}{2}\right) + 1 = \frac{1}{4}$$