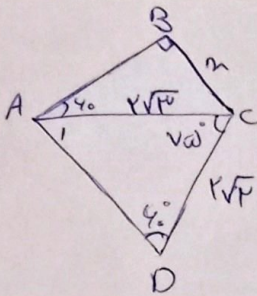


$$\begin{aligned}\hat{A} + \hat{B} + \hat{C} &= 180^\circ \\ \hat{B} + \hat{C} &= 140^\circ \\ \hat{A} &= 40^\circ\end{aligned}$$

$$\frac{BC}{\sin \hat{A}} = \frac{AC}{\sin \hat{B}} \Rightarrow \frac{10}{\frac{\sqrt{10}}{3}} = \frac{10\sqrt{4}}{3} = \frac{10\sqrt{4}}{\sin \hat{B}} \Rightarrow \frac{10\sqrt{4}}{3} = \frac{10\sqrt{4}}{3 \sin \hat{B}}$$

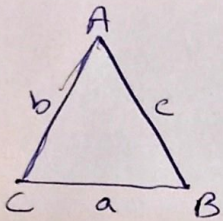
$$\sin \hat{B} = \frac{\sqrt{10}}{3} \Rightarrow \hat{B} = 40^\circ, \hat{C} = 100^\circ$$



$$\hat{A}_1 = 180 - 40 - 100 = 40^\circ$$

$$\frac{CD}{\sin 40^\circ} = \frac{AC}{\sin 40^\circ} \Rightarrow \frac{\frac{\sqrt{10}}{3}}{\sin 40^\circ} = \frac{AC}{\sin 40^\circ} \Rightarrow \frac{\sqrt{10}}{3} = AC \Rightarrow AC = \frac{\sqrt{10}}{3}$$

$$\sin 40^\circ = \frac{\sqrt{10}}{3} = \frac{n}{\frac{\sqrt{10}}{3}} \Rightarrow n = \frac{\sqrt{10} \times \sqrt{10}}{3} = \frac{10}{3}$$

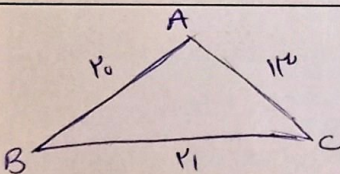


$$\frac{(b^2 - r)(\sin \hat{C})}{\sin \hat{B}} = \frac{c}{1} \Rightarrow \frac{\sin \hat{C}}{\sin \hat{B}} = \frac{c}{b^2 - r} \Rightarrow \frac{\sin \hat{C}}{c} = \frac{\sin \hat{B}}{b^2 - r}$$

$$\frac{c}{\sin \hat{C}} = \frac{b^2 - r}{\sin \hat{B}} \Rightarrow \frac{c}{\sin \hat{C}} = \frac{b}{\sin \hat{B}} \Rightarrow b^2 - r = b$$

$$b^2 - b - r = 0 \quad (b-1)(b+1) = 0 \Rightarrow b = -1 \text{ (غیرممکن)}$$

$$\Rightarrow \underline{b = 2} \text{ (قوی)}$$



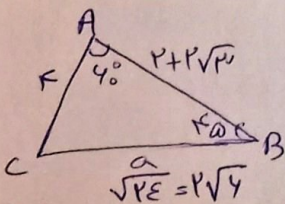
$$AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos \hat{B}$$

$$14^2 = 10^2 + 11^2 - 2(10)(11)\cos \hat{B}$$

$$\cos \hat{B} = \frac{44}{110} = \frac{4}{10}$$

$$\sin^2 \hat{B} + \cos^2 \hat{B} = 1 \quad \frac{16}{100} + \sin^2 \hat{B} = 1 \Rightarrow \sin \hat{B} = \frac{4}{10}$$

عشق
زاویه
است
عشق

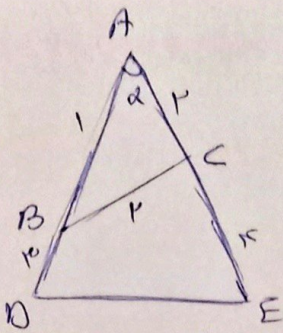


$$a^2 = 14^2 + (10+11)^2 - 2(14)(21)\cos \frac{1}{2} \Rightarrow$$

$$a^2 = 14^2 + 441 - 2(14)(21)\cos \frac{1}{2} = 25 \Rightarrow a = \sqrt{25}$$

$$\frac{a}{\sin 40^\circ} = \frac{10}{\sin \hat{B}} \Rightarrow \frac{\sqrt{25}}{\sin 40^\circ} = \frac{10}{\sin \hat{B}} \Rightarrow \sin \hat{B} = \frac{\sqrt{25}}{10} \Rightarrow \hat{B} = 40^\circ$$

$$\hat{C} = 180 - 40 - 40 = 100^\circ$$



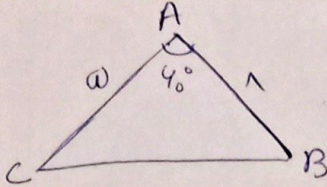
$$BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos\alpha$$

$$1 = 1 + 1 - 2 \times 1 \times 1 \cos\alpha \Rightarrow \cos\alpha = \frac{1}{2}$$

$$DE^2 = AD^2 + AE^2 - 2(AD)(AE)\cos\alpha$$

$$DE^2 = \underbrace{1^2 + 1^2}_{=2} - 2 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \Rightarrow DE^2 = \frac{3}{2}$$

$$DE = \sqrt{\frac{3}{2}}$$



$$BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos 40^\circ$$

$$BC^2 = \underbrace{10^2 + 1^2}_{=101} - \underbrace{2 \times 10 \times 1 \times \frac{1}{2}}_{=10} = 91 \Rightarrow BC = \sqrt{91}$$

$$S = \frac{1}{2} \times AB \times AC \times \sin 40^\circ =$$

$$\frac{1}{2} \times 10 \times 1 \times \frac{\sqrt{10}}{2} = \frac{10\sqrt{10}}{2}$$

$$\frac{(b+c)^2 - a^2}{bc \cos \hat{A} + bc} = \frac{b^2 + c^2 + 2bc - a^2}{bc \cos \hat{A} + bc} = \frac{b^2 + c^2 + 2bc - a^2 - 2bc \cos \hat{A} - 2bc}{bc \cos \hat{A} + bc}$$

$$\xrightarrow{\text{نقص } 2bc} \frac{b^2 + c^2 - 2bc \cos \hat{A} + 2bc - 2bc - a^2}{bc \cos \hat{A} + bc}$$

(نقص 2bc من البسط والفرق)

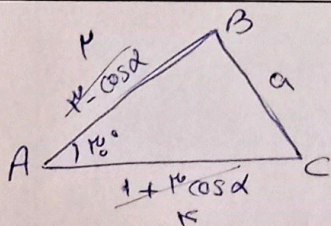
$$\frac{a^2 - a^2}{bc \cos \hat{A} + bc} = 0$$

$$\frac{a^2 + b^2 - c^2}{a+b-c} = a^2 \Rightarrow a^2 + b^2 - c^2 = a^2 + b^2 - ca^2$$

$$(b-c)(b^2 + c^2 + bc) = a^2(b-c)$$

$$a^2 = b^2 + c^2 + bc \quad a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Rightarrow bc = -2bc \cos A \Rightarrow \cos A = -\frac{1}{2}$$



$$S = \frac{1}{2} \times \frac{1}{2} \times (1 - \cos \alpha)(1 + \cos \alpha) = \frac{1}{2} \quad \cos \alpha = \frac{1}{2}$$

$$\Rightarrow (1 - t)(1 + t) = \frac{1}{2} \quad -t^2 + 1 = \frac{1}{2} \Rightarrow t^2 = \frac{1}{2} \Rightarrow t = \frac{1}{\sqrt{2}} \quad \text{أو } t = -\frac{1}{\sqrt{2}}$$

$$a+b+c=0$$

$$-1 \leq \cos \alpha \leq 1$$

$$a^2 = AB^2 + AC^2 - 2(AB)(AC)\cos 120^\circ$$

$$a^2 = 1 + 1 - 2 \times 1 \times 1 \times \left(-\frac{1}{2}\right) = 2 \Rightarrow a = \sqrt{2}$$