

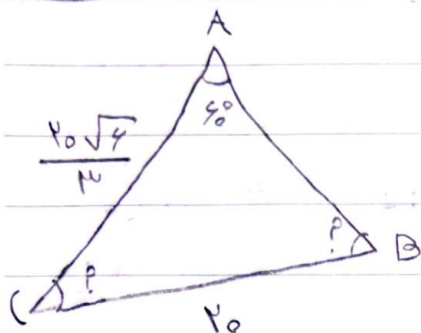
تکلیف شماره ۱۳

نیایش زندی

به نام خدا

1

با توجه به قضیه سینوس داریم:



$$\frac{BC}{\sin A} = \frac{AC}{\sin B} \rightarrow \frac{20}{\frac{\sqrt{3}}{2}} = \frac{20\sqrt{3}}{\sin B}$$

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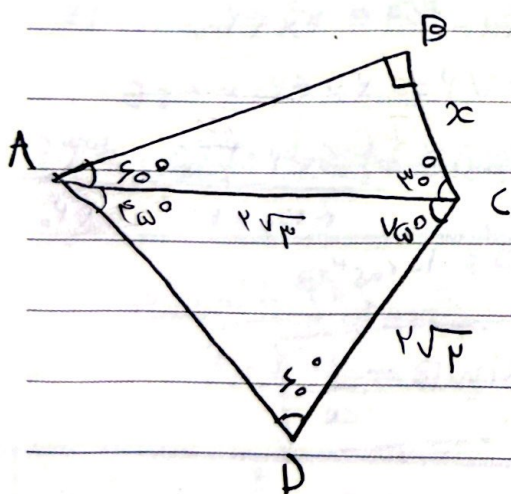
$$\sin B = \frac{20\sqrt{3}}{\frac{20\sqrt{3}}{\sin B}} = \frac{\sqrt{12}}{6} = \frac{\sqrt{4}}{\sqrt{3}} = \frac{\sqrt{2}}{2}$$

$$\sin B = \frac{\sqrt{2}}{2} \Rightarrow B = 45^\circ$$

$$180 - 60 - 45 = 75^\circ$$

$$B = 45^\circ \text{ و } C = 75^\circ$$

2



$$\frac{DC}{\sin 60^\circ} = \frac{AC}{\sin 30^\circ}$$

$$\frac{2\sqrt{3}}{\frac{\sqrt{3}}{2}} = \frac{AC}{\frac{1}{2}} \rightarrow \frac{4}{1} = \frac{AC}{\frac{1}{2}}$$

$$AC = \frac{\sqrt{3}}{2} \times 4 = 2\sqrt{3}$$

$$\sin 30^\circ = \frac{\sqrt{3}}{2} = \frac{x}{AC} = \frac{x}{2\sqrt{3}}$$

$$\frac{x}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$$x = \frac{\sqrt{3} \times 2\sqrt{3}}{2} = 3$$

طبق قانون سینوس ها

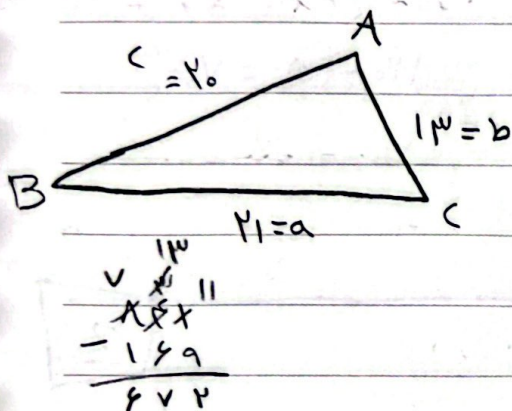
$$\frac{b}{\sin B} = \frac{c}{\sin C} \rightarrow \frac{\sin C}{\sin B} = \frac{c}{b} \quad I$$

$$\frac{\sin C}{\sin B} = \frac{c}{b^2 - 1} \quad II \quad I, II \Rightarrow \frac{c}{b} = \frac{c}{b^2 - 1}$$

$$b^2 - 1 = b \quad b^2 - b - 1 = 0 \quad (b-2)(b+1) = 0 \quad b = -1 \quad \boxed{b=2}$$

غلق

طبق قانون کسینوس ها



$$b^2 = a^2 + c^2 - 2(ac \cos B)$$

$$13^2 = 2^2 + 20^2 - 2(2 \times 20 \times \cos B)$$

$$169 = 4 + 400 - 2(40 \cos B)$$

$$169 = 404 - 2(40 \cos B)$$

$$169 - 404 = -2 \times 40 \times \cos B$$

$$235 = 2 \times 40 \times \cos B$$

$$\cos B = \frac{235}{80} = \frac{47}{16}$$

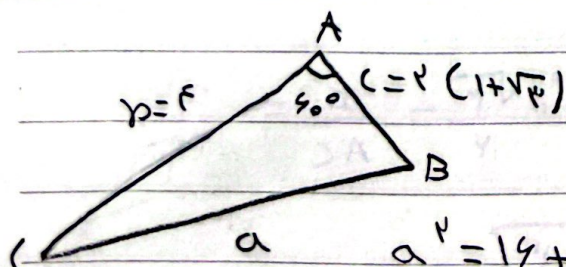
$$= \frac{11}{16} = \frac{\cos B}{\frac{1}{16}} = \frac{1}{16} = \frac{1}{16}$$

$$\sin B = 1 - \cos B$$

$$\sin^2 B = 1 - \frac{16}{256}$$

$$\sin^2 B = \frac{9}{256}$$

$$\sin B = \frac{3}{16}$$



$$a^2 = b^2 + c^2 - 2(bc \cos A)$$

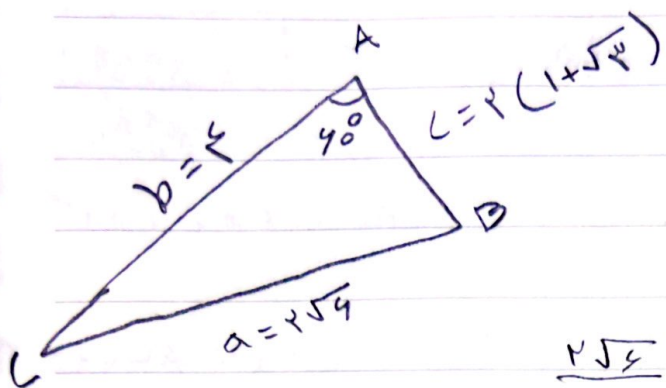
$$a^2 = 14^2 + 4^2 - 2(4 \times 2(1+\sqrt{3}) \cos 60^\circ)$$

$$a^2 = 196 + 16 - 2(8(1+\sqrt{3}) \times \frac{1}{2})$$

$$a^2 = 212 + 16\sqrt{3} - (8(1+\sqrt{3}) \times \frac{1}{2}) = 212 + 16\sqrt{3} - 4 - 4\sqrt{3} =$$

$$= 208 + 12\sqrt{3} \quad a = \sqrt{208 + 12\sqrt{3}} = 2\sqrt{52 + 3\sqrt{3}}$$

Q. No. 131



$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

(.)

$$\frac{\frac{f\sqrt{4}}{1}}{\frac{f\sqrt{4}}{f}} = \frac{f}{\sin B}$$

$$\frac{f\sqrt{4}}{\sqrt{4}} = \frac{f}{\sin B}$$

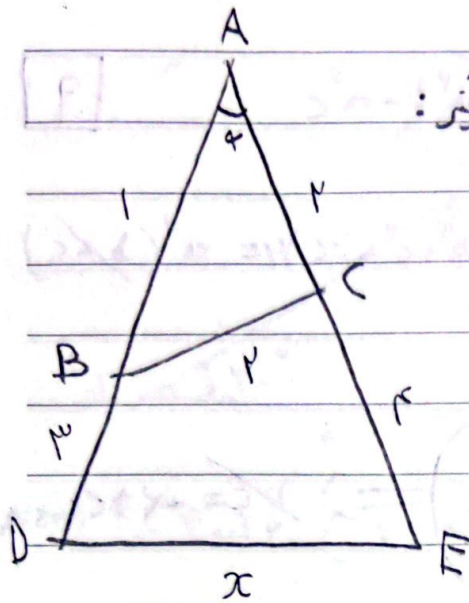
$$f\sqrt{4} = \frac{f}{\sin B}$$

$$\sin B = \frac{\sqrt{4}}{4} \Rightarrow \boxed{B = 40^\circ}$$

$$\hat{A} + \hat{B} + \hat{C} = 180$$

$$90 + 40 + \hat{C} = 180$$

$$\boxed{\hat{C} = 50}$$



در مثلث ABC قضیه کسینوس ها را برای کثیر:

$$BC^2 = AB^2 + AC^2 - 2 \times AB \times AC \times \cos \varphi$$

$$4 = 1 + 4 - (2 \times 1 \times 2 \times \cos \varphi)$$

$$4 = 5 - (4 \cos \varphi) \rightarrow 4 \cos \varphi = 1 \quad \cos \varphi = \frac{1}{4}$$

حالا در مثلث ADE قضیه کسینوس ها را داریم:

$$DE^2 = AD^2 + AE^2 - (2 \times AD \times AE \times \cos \varphi)$$

$$x^2 = 1^2 + 2^2 - (2 \times 1 \times 2 \times \frac{1}{4})$$

$$x^2 = 5 - 1 \quad x^2 = 4 \quad x = \sqrt{4} = 2$$

$$BC^2 = AC^2 + AB^2 - (2 \times AC \times AB \times \cos 60^\circ)$$

الف) از قانون کسینوس ها استفاده می کنیم:

$$BC^2 = 2^2 + 9 - (2 \times 2 \times 3 \times \frac{1}{2})$$

$$BC^2 = 19 - 6 = 13 \quad BC = \sqrt{13} = \sqrt{13}$$

$$S_{ABC} = \frac{1}{2} \times AC \times AB \times \sin \hat{A} = \frac{1}{2} \times 2 \times 3 \times \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}$$

$$\frac{b^2 + c^2 + 2bc - a^2}{2c(\cos A + 1)} = \frac{2bc \cos A + 2bc}{2c(\cos A + 1)}$$

$$a^2 = b^2 + c^2 - (2 \times b \times c \times \cos \varphi) \rightarrow b^2 + c^2 - a^2 = 2bc \cos A$$

$$\frac{2bc(\cos A + 1)}{2c(\cos A + 1)} = \sqrt{13}$$

$$\frac{a^3 + b^3 - c^3}{a + b - c} = a^2$$

$$\cancel{a^3} + b^3 - c^3 = \cancel{a^3} + a^2 b - a^2 c$$

9

$$b^3 - c^3 = a^2 b - a^2 c$$

$$(b - c)(b^2 + bc + c^2) = a^2(b - c)$$

$$a^2 = b^2 + c^2 + bc$$

بر اساس
قضیه کسینوس
ما داریم

$$\left. \begin{array}{l} \text{این ها را برابر قرار می دهیم} \\ \end{array} \right\} = -1 \quad \cancel{bc} = -2bc \cos A$$

$$\rightarrow a^2 = b^2 + c^2 - 2bc \cos A$$

$$-2 \cos A = 1$$

$$\cos A = -\frac{1}{2}$$

10

$$\frac{1}{2} \times AB \times AC \times \sin 120^\circ = S_{ABC} = 2$$

$$\frac{1}{2} \times (3 - \cos \theta) \times (1 + 3 \cos \theta) \times \underbrace{\sin 120^\circ}_{\frac{1}{2}} = 2$$

$$(3 - \cos \theta)(1 + 3 \cos \theta) = 8$$

$$3 + 9 \cos \theta - \cos \theta - 3 \cos^2 \theta = 8$$

$$1 \cos \theta$$

$$3 \cos^2 \theta - 1 \cos \theta + 1 = 0$$

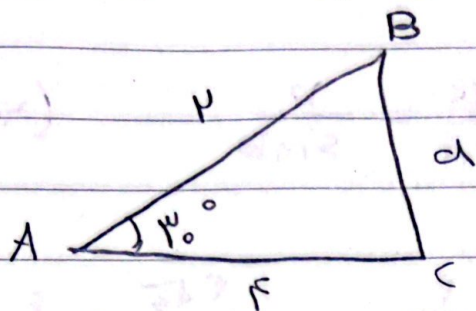
$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{1 - 4(3)(1)}}{6} = \frac{1 \pm 2}{6}$$

$$\cos \theta \begin{cases} \frac{3}{2} \rightarrow \text{غیر قابل است زیرا} \\ -1 \leq \cos \theta \leq 1 \end{cases}$$

این جواب برای پذیر نیست ✓

ادامه واد ← دانستیم $\cos \theta = 1$ الفون دایر

حال برای پیدا کردن a از قانون
کسینوس ها استفاده می کنیم.



$$a^2 = AB^2 + AC^2 - (2 \times AB \times AC \times \cos 30^\circ)$$

$$a^2 = 4 + 16 - (2 \times 2 \times 4 \times \underbrace{\cos 30^\circ}_{\frac{\sqrt{3}}{2}}) = 20 - 8\sqrt{3}$$