فصل ... تکلیف شماره ۱۳ — دوازدهم A — نگین ... — ۲۵

۱-

$\frac{b}{\sin B}=\frac{a}{\sin A}=\frac{c}{\sin C}$ *

$BC=20$

$AC=\frac{20\sqrt6}{3}$

$B+C=120$ → (مجموع زوایای داخلی مثلث ۱۸۰ درجه است) → $A+B+C=180$, $A=60$

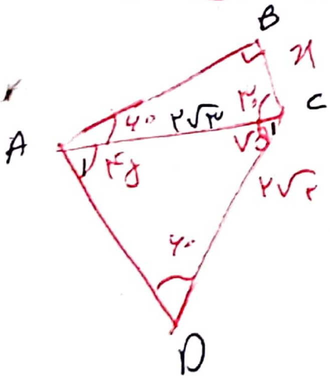
$\frac{\frac{20\sqrt6}{3}}{\sin B}=\frac{20}{\frac{\sqrt3}{2}}$

$\sin B=\frac{\frac{20\sqrt6}{3}\times\frac{\sqrt3}{2}}{20}=\frac{3\sqrt2}{6}=\frac{\sqrt2}{2}$ → $B=45$ ✓

$B+C=120$ → $C=75$ ✓

(۲)

۲- طبق رابطه *



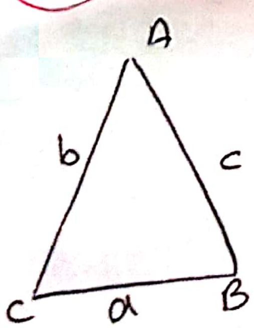
(مجموع زوایای داخلی مثلث) $A_1+C_1+D=180 \Rightarrow A_1=45°$

$\frac{2\sqrt2}{\frac{\sqrt2}{2}}=\frac{AC}{\frac{\sqrt3}{2}} \Rightarrow AC=2\sqrt3$

$\frac{2\sqrt3}{1}=\frac{x}{\frac{\sqrt3}{2}} \Rightarrow x=\frac{\sqrt3}{2}\times2\sqrt3 \Rightarrow x=3$ ✓

(۲)

۳-



$\frac{\sin C}{\sin B}=\frac{c}{b}$

$(b^2-2)\frac{c}{b}=c$ → چون ۴۵° است می‌توانیم از طرفین خط بزنیم چون ... هم هست

$b^2-2=b$

$b^2-b-2=0$

$(b-2)(b+1)=0$

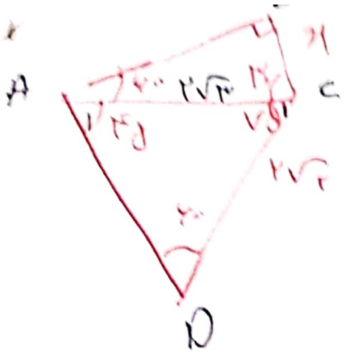
$b-2=0$, $b=2$ قابل قبول

$b+1=0$, $b=-1$ غیرقابل قبول

$b=2$ ✓

(۲)

۲- طبق روابط *

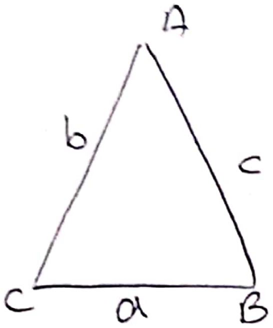


$A_1 + C_1 + D = 180 \Rightarrow A_1 = 45^\circ$

$$\frac{2\sqrt{2}}{\frac{\sqrt{2}}{2}} = \frac{AC}{\frac{\sqrt{3}}{2}} \Rightarrow AC = 2\sqrt{3}$$

$$\frac{2\sqrt{3}}{1} = \frac{x}{\frac{\sqrt{3}}{2}} \Rightarrow x = \frac{\sqrt{3}\times 2\sqrt{3}}{2} \Rightarrow x = 3$$

۳-



$$\frac{\sin C}{\sin B} = \frac{c}{b}$$

$(b^2-2)\frac{c}{b} = c$ — چون c مثبت است می‌توانیم از طرفین خط بزنیم

$b^2 - 2 = b$

$b^2 - b - 2 = 0$

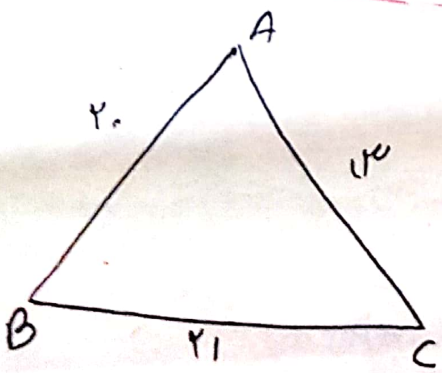
$(b-2)(b+1) = 0$

$b-2=0 \Rightarrow b=2$ قابل قبول

$b+1=0 \Rightarrow b=-1$ غیرقابل قبول

$b = 2$

۴-



$a^2 = b^2 + c^2 - 2bc\cos A$ *1

$(13)^2 = 21^2 + 20^2 - 2\times 20\times 21\times \cos B$

$169 = 441 + 400 - 840\cos B$

$-672 = -840\cos B$

$96 = 120\cos B \rightarrow \cos B = \frac{96}{120} = \frac{8}{10}$ ✓

$\cos^2 B + \sin^2 B = 1$

$\Rightarrow \sin B > 0$

$\Rightarrow \sin B = \frac{6}{10}$ ✓

۲

۵- طبق رابطه ۱ در سوال قبل

الف)

$a^2 = 16 + (2+2\sqrt{3})^2 - 2(2+2\sqrt{3})4\cos 60$

$a^2 = 16 + 4 + 12 + 8\sqrt{3} - 8 - 4\sqrt{3} - 4\sqrt{3}$

$a^2 = 24 + 4\sqrt{3} - 4\sqrt{3} \rightarrow a = \sqrt{24 + 4\sqrt{3} - 4\sqrt{3}} \Rightarrow a = \sqrt{24} = 2\sqrt{6}$ ✓

۲

ب)

$\frac{c}{\sin C} = \frac{b}{\sin B} = \frac{a}{\sin A} \Rightarrow \frac{2(1+\sqrt{3})}{\sin C} = \frac{4}{\sin B} = \frac{2\sqrt{6}}{\frac{\sqrt{3}}{2}} = 4\sqrt{2}$

$\sin B = \frac{\sqrt{2}}{2} \rightarrow B = 45$ ✓

$C = 180 - 60 - 45 = 75$ ✓

۶-

$BC^2 = AB^2 + AC^2 - 2\times AB \times AC \cos\alpha \Rightarrow 4 = 4 + 1 - 2\times 2\times 1\times \cos\alpha$

$-1 = -4\cos\alpha \Rightarrow \cos\alpha = \frac{1}{4}$ ✓

$DE^2 = AD^2 + AE^2 - 2AD\,AE\cos\alpha \Rightarrow DE^2 = 16 + 36 - 2\times 4\times 6\times \frac{1}{4} = 40$

$DE = \sqrt{40} = 2\sqrt{10}$ ✓

۲

۷-

$\sin 60 = \frac{\sqrt{3}}{2} \qquad \cos 60 = \frac{1}{2}$

الف) طبق رابطه ۱،

$BC^2 = 8^2 + 5^2 - 2\times 8\times 5\times \frac{1}{2}$

$BC^2 = 89 - 40 = 49 \rightarrow$ $BC = 7$

۶-

$BC^2 = AB^2 + AC^2 - 2 \times AB \times AC \cos\alpha \Rightarrow 4 = 4 + 1 - 2 \times 2 \times 1 \times \cos\alpha$

$-1 = -4\cos\alpha \Rightarrow \cos\alpha = \frac{1}{4}$

$DE^2 = AD^2 + AE^2 - 2AD \cdot AE \cos\alpha \Rightarrow DE^2 = 16 + 36 - 2 \times 4 \times 6 \times \frac{1}{4} = 40$

$DE = \sqrt{40} = 2\sqrt{10}$

۷-

$\sin 60 = \frac{\sqrt{3}}{2}$ $\cos 60 = \frac{1}{2}$

الف) طبق رابطه x_1

$BC^2 = 8^2 + 5^2 - 2 \times 8 \times 5 \times \frac{1}{2}$

$BC^2 = 89 - 40 = 49 \rightarrow$ $BC = 7$ ✓

(۲)

ب)

$S_{ABC} = AB \times AC \times \sin 60 \times \frac{1}{2} = 8 \times 5 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = 40 \times \frac{\sqrt{3}}{4} = 10\sqrt{3}$ ✓

۸-

$a^2 = b^2 + c^2 - 2bc\cos A$

$(b+c)^2 - a^2 = b^2 + c^2 + 2bc - b^2 - c^2 + 2bc\cos A = 2bc(1+\cos A)$

(۲)

$-1 \leq \cos A \leq 1$

$0 \leq 1 + \cos A \leq 1$

$\frac{(b+c)^2 - a^2}{bc(\cos A + 1)} = \frac{2bc(1+\cos A)}{bc(1+\cos A)} = 2$ ✓

[illegible]
امکان پذیر نیست

۹-

$a^3 + a^2b - a^2c = a^3 + b^3 - c^3$

$a^2b - b^3 = a^2c - c^3$

$b(a^2 - b^2) = c(a^2 - c^2)$

$b^3 - c^3 = a^2b - a^2c$

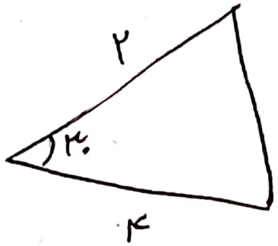
$(b-c)(b^2+c^2+bc) = (b-c)(b^2+c^2-2bc\cos A)$

$\cos A = -\frac{1}{2} \to \hat{A} = 120^\circ$

۲

۱۰- طبق رابطه

$S_{ABC} = \frac{1}{2} \times b \times c \times \sin 30$



$2 = \frac{1}{4} \times (3-\cos\theta)(1+3\cos\theta)$

$(3-\cos\theta)(1+3\cos\theta) = 8$

$3 + 9\cos\theta - \cos\theta - 3\cos^2\theta = 8$

$-3\cos^2\theta + 8\cos\theta - 5 = 0$ → روش ac / روش ...

$\cos^2\theta + 8\cos\theta + 15 = 0$

$(\cos\theta + 3)(\cos\theta + 5) = 0$

$\cos\theta + 3 = 0$, $\cos\theta = \frac{-3}{-3} = 1$

$\cos\theta + 5 = 0$, $\cos\theta = \frac{-5}{-3}$ غیرقابل قبول

$-1 \le \sin\theta \le 1$

۲

$a^2 = 2^2 + 4^2 - 2 \times 2 \times 4 \times \frac{\sqrt{3}}{2}$

$\to a^2 = 20 - 8\sqrt{3}$

طبق رابطه ۱*