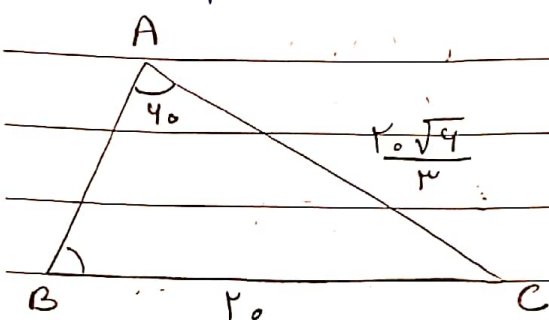


دوار در سمت راست A. ۱۲۰°

میدانهای مثلث (مستوی)

$$\hat{B} + \hat{C} = 120^\circ \Rightarrow \hat{A} = 40^\circ$$

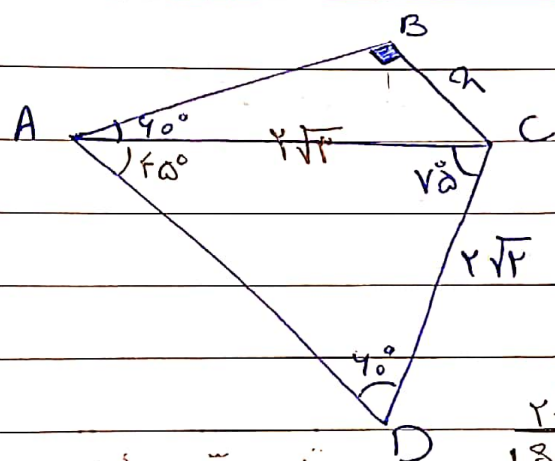


$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{20}{\frac{\sqrt{3}}{2}} = \frac{\frac{20\sqrt{4}}{3}}{\sin B}$$

$$\frac{20}{\sqrt{3}} = \frac{20\sqrt{4}}{3\sin B} \Rightarrow 3\sin B = \frac{20\sqrt{4} \times \sqrt{3}}{20} \Rightarrow \sin B = \frac{\sqrt{12}}{4}$$

$$\sin B = \frac{3\sqrt{3}}{4} = \frac{\sqrt{3}}{2} \text{ و } B \Rightarrow \hat{B} = 60^\circ \Rightarrow C = 120 - 40 = 80^\circ$$

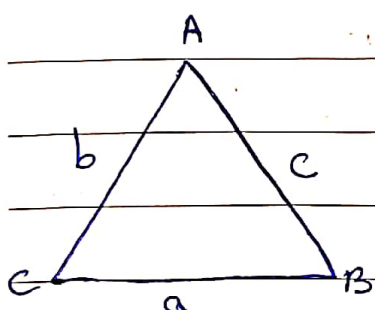


$$\frac{20\sqrt{3}}{\sin 80} = \frac{AC}{\sin 40} \Rightarrow AC = \frac{20\sqrt{3} \sin 40}{\sin 80}$$

$$\frac{20\sqrt{3}}{\frac{\sqrt{3}}{2}} = \frac{AC}{\frac{\sqrt{3}}{2}} \Rightarrow AC = 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$$

$$\frac{20\sqrt{3}}{1 \sin 90} = \frac{a}{\frac{\sin 40}{\sqrt{3}}} \Rightarrow a = 20\sqrt{3} \times \frac{\sqrt{3}}{2} = 30$$

$$\Rightarrow a = 30$$



$$(b^2 - r) \frac{\sin \hat{C}}{\sin \hat{B}} = c$$

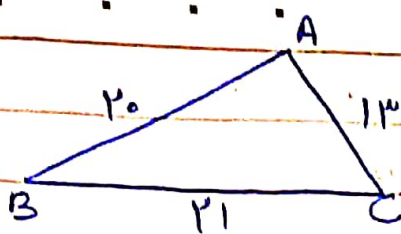
$$\frac{c}{\sin C} = \frac{b}{\sin B} \Rightarrow \frac{\sin \hat{C}}{\sin \hat{B}} = \frac{c}{b}$$

$$\Rightarrow (b^2 - r) \times \frac{c}{b} = c \Rightarrow b^2 - r = b$$

$$b^2 - b - r = 0$$

$$(b - r)(b + 1) = 0$$

$$\begin{cases} b = r \checkmark \\ b = -1 \times \end{cases}$$

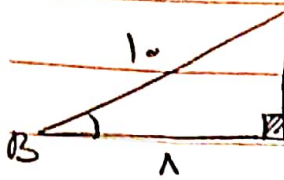


$$\sin \hat{B} = ?$$

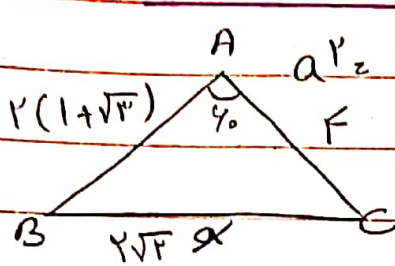
$$10^2 = 10^2 + 10^2 - 2(10)(10)(\cos \hat{B})$$

$$100 = 100 + 100 - 200 \cos \hat{B}$$

$$200 \cos \hat{B} = 100 \Rightarrow \cos \hat{B} = \frac{1}{2}$$



$$\Rightarrow \sin B = \frac{4}{10}$$



$$a^2 = 10^2 + 10^2 - 2(10)(10) \cos \hat{A}$$

$$a^2 = 10^2 + 10^2 - 200 \cos 60^\circ$$

$$a^2 = 100 + 100 - 100 = 100$$

$$\Rightarrow a = 10$$

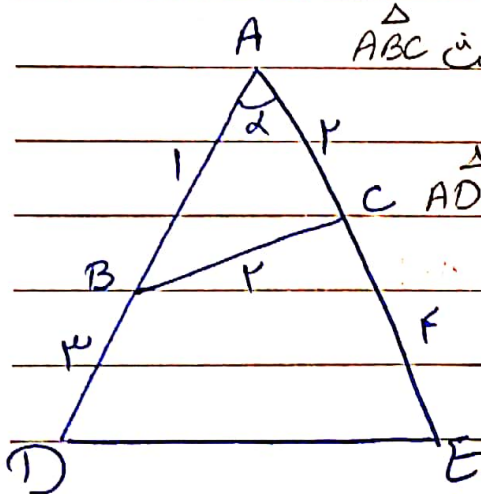
$$10 + 10 - 20 \cos \hat{B} = 10$$

$$10 + 10 - 20 \cos \hat{B} = 10$$

$$10 + 10 - 20 \cos \hat{B} = 10$$

$$10 + 10 - 20 \cos \hat{B} = 10 \Rightarrow \cos \hat{B} = \frac{1}{2} \Rightarrow \hat{B} = 60^\circ$$

$$\Rightarrow \hat{C} = 180^\circ - (100^\circ + 60^\circ) = 20^\circ$$



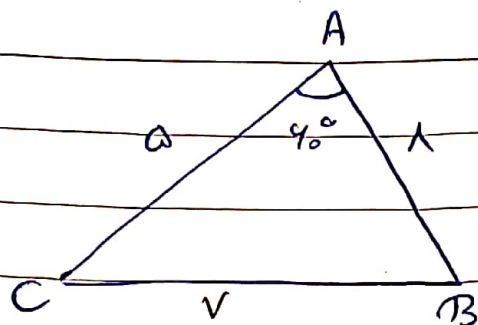
$$\triangle ABC \text{ قائم } \Rightarrow 10^2 = 10^2 + 10^2 - 2(10)(10) \cos A$$

$$10 \cos A = 1 \Rightarrow \cos A = \frac{1}{10}$$

$$\triangle ADE \text{ قائم } \Rightarrow DE^2 = 10^2 + 10^2 - 2(10)(10) \cos A$$

$$DE^2 = 100 + 100 - 100 = 100 \Rightarrow DE = 10$$

$$DE = 10$$



BC = ? (✓)

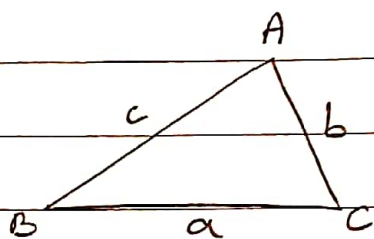
$$BC^2 = 2^2 + 1^2 - 2(2)(1)\cos 40^\circ$$

$$BC^2 = 2^2 + 1^2 - 2$$

$$BC^2 = 1 \Rightarrow BC = 1$$

$$S_{\triangle ABC} = \frac{1}{2} \times 2 \times 1 \times \sin 40^\circ = \sin 40^\circ$$

$S_{\triangle ABC} = ?$ (✓)



$$\frac{(b+c)^2 - a^2}{bc(\cos A + 1)}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$(b+c)^2 - a^2 =$$

$$b^2 + c^2 + 2bc - b^2 - c^2 + 2bc \cos A = 2bc(\cos A + 1)$$

$$\Rightarrow \frac{2bc(\cos A + 1)}{bc(\cos A + 1)} = 2$$

$$\frac{a^2 + b^2 - c^2}{a + b - c} = a$$

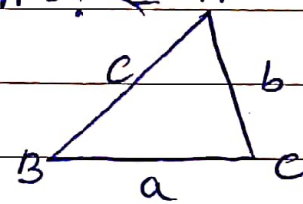
$\cos A = ?$ (✓)

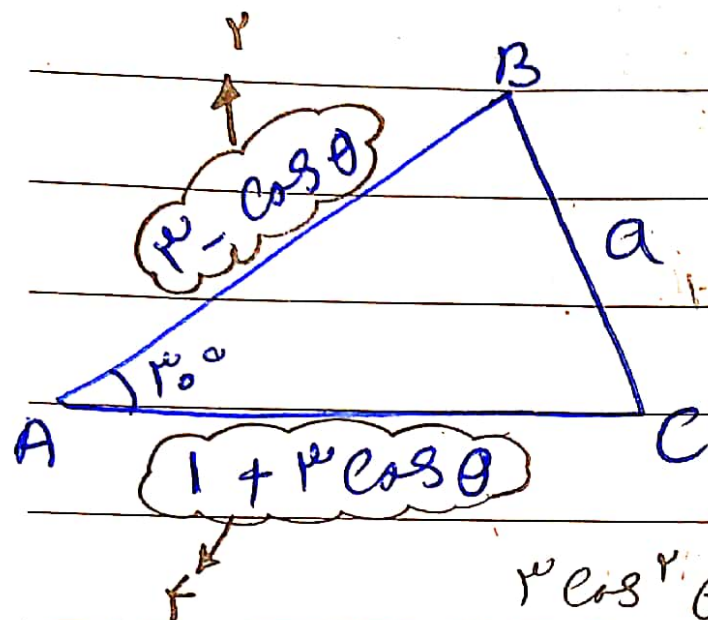
$$a^2 + b^2 - c^2 = a^2 + ab - ac$$

$$(b-c)(b^2 + bc + c^2) = a^2(b-c)$$

$$a^2 = b^2 + c^2 + bc$$

$$\left. \begin{array}{l} a^2 = b^2 + c^2 + bc \\ a^2 = b^2 + c^2 - 2bc \cos A \end{array} \right\} \Rightarrow \cos A = -\frac{1}{2}$$





$$S_{ABC} = r \rightarrow a^r = ? \quad -1.$$

$$S_{ABC} = \frac{1}{r} (1 + r \cos \theta) (r - \cos \theta) \times \frac{1}{r} = r$$

$$r - \cos \theta + r \cos \theta - r \cos^2 \theta = 1$$

$$r \cos^2 \theta - 1 \cos \theta + 1 = 0$$

$$* \cos^2 \theta - 1 \cos \theta + 1 = 0 \Rightarrow (\cos \theta - r)(\cos \theta - 1) = 0$$

$$\cos \theta = \frac{r}{1} \quad \checkmark \quad \cos \theta = \frac{1}{r} \quad \times$$

$$-1 \leq \cos \theta \leq 1 \quad \text{و چون}$$

$$a^r = 1 + r - r(r)(r) \cos \theta_0$$

این قضیه کسینوس است

$$a^r = r_0 - 1 + r \times \frac{\sqrt{r}}{r} \Rightarrow a^r = r_0 - 1 \sqrt{r}$$