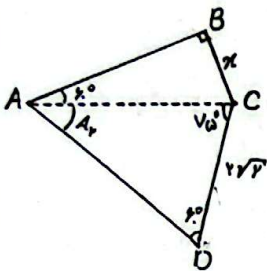


$$\hat{A} + \hat{B} + \hat{C} = 180^\circ \Rightarrow \hat{A} = 40^\circ$$

$$\text{طبق سینوسها: } \frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{2}{\sin 40^\circ} = \frac{1}{\sin B} \Rightarrow \frac{2}{\frac{\sqrt{2}}{2}} = \frac{1}{\sin B} \Rightarrow \frac{4}{\sqrt{2}} = \frac{1}{\sin B} \Rightarrow \sin B = \frac{\sqrt{2}}{4}$$

$$\Rightarrow \sin B = \frac{\sqrt{2}}{4} \Rightarrow \hat{B} = 45^\circ, \hat{C} = 180^\circ - 40^\circ - 45^\circ = 95^\circ$$

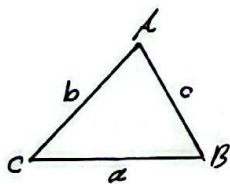
با توجه به اینکه $\hat{B} < \hat{C}$ پس $\hat{B} = 45^\circ$ صحیح است.



$$90^\circ + 40^\circ + \hat{A}_r = 180^\circ \Rightarrow \hat{A}_r = 50^\circ \Rightarrow \sin 50^\circ = \frac{\sqrt{2}}{2}$$

$$\frac{2\sqrt{2}}{\frac{\sqrt{2}}{2}} = 4 = \frac{\overline{AC}}{\sin 50^\circ} = \frac{\overline{AC}}{\frac{\sqrt{2}}{2}} \Rightarrow \overline{AC} = 2\sqrt{2}$$

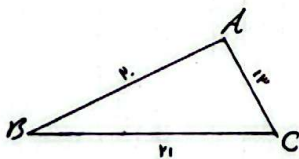
$$\frac{\overline{AC}}{\sin B} = \frac{x}{\sin 40^\circ} \Rightarrow \frac{2\sqrt{2}}{1} = \frac{x}{\frac{\sqrt{2}}{2}} \Rightarrow x = 2$$



$$\text{طبق قوسینوسها: } \frac{c}{\sin C} = \frac{b}{\sin B} \Rightarrow \sin B = \frac{b \sin C}{c}$$

$$(b^2 - c^2) \frac{\sin C}{b \sin C} = c \Rightarrow (b^2 - c^2) \frac{c}{b} = c \Rightarrow b^2 - b - 2 = 0$$

$$a + c = b \Rightarrow b = \sqrt{a^2 + c^2} \rightarrow \text{غلط}$$



$$\text{طبق قوسینوسها: } (11)^2 = (7)^2 + (11)^2 - (7)(7)(11)(\cos \hat{B})$$

$$\Rightarrow 121 = 49 + 121 - 154 \cos \hat{B} \Rightarrow \cos \hat{B} = \frac{49}{154} = \frac{7}{22}$$

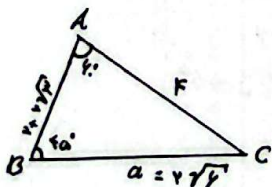
$$\sin^2 B = 1 - \cos^2 B \Rightarrow \sin^2 B = 1 - \frac{49}{484} = \frac{435}{484} \Rightarrow \sin B = \pm \frac{\sqrt{435}}{22}$$

به مقدار منفی غیر قابل قبول!

$$\Rightarrow \sin \hat{B} = \frac{\sqrt{435}}{22}$$

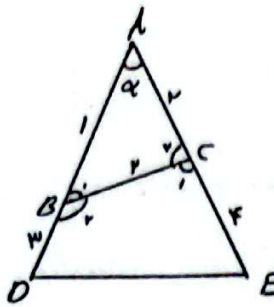
$$\text{طبق قوسینوسها: } a^2 = 14 + 14 + 14\sqrt{3} - 14 - 14\sqrt{3} = 24 \Rightarrow a = \sqrt{24} \Rightarrow a = 2\sqrt{6}$$

غلط



$$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{2\sqrt{6}}{\frac{\sqrt{2}}{2}} = \frac{4}{\sin B} \Rightarrow \sin B = \frac{\sqrt{2}}{2} \Rightarrow \hat{B} = 45^\circ$$

$$\hat{C} = 95^\circ$$



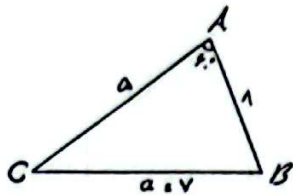
ΔABC مثلث متساوی الساقین $\Rightarrow B_1 = \alpha$ / ΔADC مثلث متساوی الساقین $\Rightarrow C_1 = 2\alpha \Rightarrow \hat{C}_2 = 180^\circ - 2\alpha$

(2)

طبق قضیه سینوس ها: $\frac{1}{\sin \alpha} = \frac{1}{\sin (180^\circ - 2\alpha)} \Rightarrow \frac{1}{\sin \alpha} = \frac{1}{\sin 2\alpha} \Rightarrow 2 \cos \alpha = 1 \Rightarrow \cos \alpha = \frac{1}{2}$

طبق قضیه کوسینوس ها: $(DE)^2 = 1^2 + 1^2 - 1^2 = 1 \Rightarrow DE = \sqrt{1} = 1$

6



$a^2 = 2\omega + 2\epsilon - 4 \Rightarrow a^2 = 49 \Rightarrow a = \sqrt{49} = 7$
 به قضیه کسینوس ها.

(2)

(ب) $S = \frac{1}{2} \times AC \times AB \times \sin \hat{A} \Rightarrow S_{ABC} = \frac{1}{2} \times \omega \times 1 \times \frac{\sqrt{3}}{2} = \frac{1 \cdot \sqrt{3}}{4}$

7

$\frac{(b+c)^2 - a^2}{bc(\cos \hat{A} + 1)} = \frac{a^2 = b^2 + c^2 - 2bc \cos \hat{A}}{bc(\cos \hat{A} + 1)} \Rightarrow \frac{b^2 + c^2 + 2bc - b^2 - c^2 + 2bc \cos \hat{A}}{bc(\cos \hat{A} + 1)} = \frac{2bc(\cos \hat{A} + 1)}{bc(\cos \hat{A} + 1)} = 2$

(2)

8

$\frac{a^3 + b^3 - c^3}{a + b - c} = a^2 \Rightarrow a^3 + b^3 - c^3 = a^2(a + b - c) \Rightarrow (b-c)(b^2 + c^2 + bc) = a^2(b-c)$

(2)

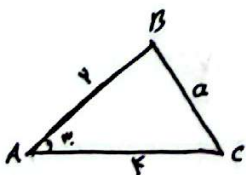
$\frac{a^2 = b^2 + c^2 - 2bc \cos \hat{A}}{b^2 + c^2 + bc} = \frac{b^2 + c^2 - 2bc \cos \hat{A}}{b^2 + c^2 + bc} \Rightarrow \cos \hat{A} = -\frac{1}{2} \rightarrow \hat{A} = 120^\circ$

9

$S_{ABC} = 2 \Rightarrow \frac{1}{2} \times (3 - \cos \theta)(1 + 3 \cos \theta) \times \frac{\sin \theta}{2} = 2 \Rightarrow \frac{1}{4} (3 - \cos \theta)(1 + 3 \cos \theta) = 2$

$\Rightarrow (3 - \cos \theta)(1 + 3 \cos \theta) = 8 \Rightarrow -3 \cos^2 \theta + 10 \cos \theta - 5 = 0 \xrightarrow{a+b+c=0} \cos \theta = \frac{1}{3} \rightarrow \theta = \cos^{-1} \frac{1}{3}$

10



طبق قضیه کسینوس ها: $a^2 = 1^2 + 1^2 - 1 \cdot 1 \cdot \sqrt{3} = 2 - \sqrt{3}$

(2)