

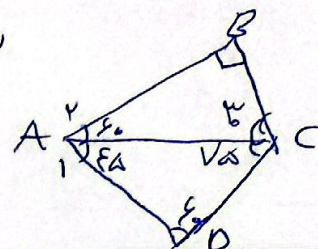
قضیه سینوس ها:

$$\frac{\sin A}{BC} = \frac{\sin B}{AC} = \frac{\sin C}{AB} \rightarrow \frac{3 \times \sin B}{20 \sqrt{2}} = \frac{\sin 60}{20} \rightarrow \sin B = \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$$

$$\rightarrow \hat{B} = 45^\circ \rightarrow C = (120 - 45 - 45) = 30^\circ$$

$$\frac{\sin A}{DC} = \frac{\sin D}{AC} \rightarrow AC = \frac{2 \times 2\sqrt{2} \times \sqrt{3}}{2 \times \sqrt{2}} = 2\sqrt{3}$$

$$\frac{\sin A}{BC} = \frac{\sin B}{AC} \rightarrow BC = 2 = \frac{2\sqrt{3} \times \sqrt{3}}{2} = 3$$



$$\frac{\sin C}{c} = \frac{\sin B}{b} = \frac{\sin A}{a} = k \Rightarrow \begin{cases} \sin C = kc \\ \sin B = kb \\ \sin A = ka \end{cases}$$

$$(b^2 - 2) \times \frac{kc}{kb} = c \rightarrow \frac{b^2 - 2}{b} = 1 \rightarrow b^2 - b - 2 = 0 \rightarrow$$

$$b_1, b_2 = \frac{1 \pm \sqrt{1+4}}{2} \rightarrow \begin{cases} b_1 = 2 \\ b_2 = -1 \end{cases}$$

قضیه کسینوس ها:

$$b^2 = a^2 + c^2 - 2ac \cos B \rightarrow 169 = 81 + 144 - (2 \times 9 \times 12 \times \cos B)$$

$$\rightarrow \cos B = \frac{9 \times 12}{2 \times 9 \times 12} = \frac{1}{2} \quad \sin^2 B + \cos^2 B = 1 \rightarrow \sin^2 B = \frac{3}{4}$$

$$\rightarrow \sin B = \pm \frac{\sqrt{3}}{2} \rightarrow \sin B = \frac{\sqrt{3}}{2}$$

$$a^2 = b^2 + c^2 - 2bc \cos A \rightarrow a^2 = 16 + 16 + 1\sqrt{3} - (2 \times 4 \times \frac{1}{2} (2 + 2\sqrt{3}))$$

$$= 32 + 1\sqrt{3} - 1\sqrt{3} = 24 \rightarrow a = 2\sqrt{6}$$

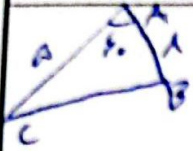
$$\frac{\sin A}{a} = \frac{\sin B}{b} \rightarrow \frac{\sqrt{3}}{2 \times 2\sqrt{6}} = \frac{\sin B}{4} \rightarrow \sin B = \frac{\sqrt{2}}{2} \rightarrow B = 45^\circ$$

$$A + B + C = 180 \rightarrow C = 180 - 45 - 60 = 75^\circ$$

$$a^2 = b^2 + c^2 - 2bc \cos A \rightarrow 8 = 1 + 9 - (2 \times 2 \times 1 \times \cos \alpha) \rightarrow \cos \alpha = \frac{1}{2}$$

$$\overline{DE}^2 = \overline{AD}^2 + \overline{AE}^2 - 2(\overline{AD} \overline{AE} \cos A) \rightarrow$$

$$DE = \sqrt{1^2 + 2^2 - (2 \times 1 \times 2 \times \frac{1}{2})} = \sqrt{2} = 2\sqrt{10}$$



$$\overline{BC} = \sqrt{1^2 + 2^2 - (2 \times 1 \times 2 \times \cos \alpha)} = \sqrt{49} = 7$$

الف) قانون کسینوس ها:

$$S_{\triangle ABC} = \frac{1}{2} \sin A \times \overline{AC} \times \overline{AB} = \frac{1}{2} \times \frac{\sqrt{3}}{2} \times 2 \times 1 = 10\sqrt{3}$$

$$\frac{(b+c)^2 - a^2}{2bc(1+\cos A)} \xrightarrow{\text{قانون کسینوس}} \frac{b^2 + c^2 + 2bc - b^2 - c^2 + 2bc \cos A}{2bc(1+\cos A)} =$$

$$\frac{2bc(1+\cos A)}{2bc(1+\cos A)} = 1$$

$$\frac{a^2 + b^2 - c^2}{a+b-c} = a^2 \rightarrow a^2 + b^2 - c^2 = a^2 + a^2 b - a^2 c \rightarrow$$

$$(b-c)(b^2 + c^2 + bc) = a^2(b-c) \xrightarrow[b-c \neq 0]{\substack{\text{ن} \\ b+c \neq 0}} b^2 + c^2 + bc = b^2 + c^2 - 2bc \cos A$$

$$\rightarrow \cos A = -\frac{1}{2} \rightarrow \hat{A} = 120^\circ$$

$$S_{\triangle ABC} = \frac{1}{2} \times \overline{AB} \times \overline{AC} \times \sin \hat{A} = \frac{1}{2} \times \frac{1}{2} (3 + 9(\cos \theta - 2\cos^2 \theta - \cos \theta)) = 2 \rightarrow$$

$$3\cos^2 \theta - 1\cos \theta + 3 = 0 \rightarrow \cos \theta_1, \cos \theta_2 = \frac{1 \pm \sqrt{1-12}}{6} \Rightarrow$$

$$\begin{cases} \cos \theta_1 = \frac{1}{2} \\ \cos \theta_2 = 1 \end{cases} \rightarrow AB = 2, AC = 3$$

قانون کسینوس ها:

$$a^2 = b^2 + c^2 - (2 \times bc \times \cos A) = 20 - 1\sqrt{3}$$