ترمیم زنجمل

۱-

$\hat{A}+\hat{B}+\hat{C}=۱۸۰^\circ \Rightarrow \hat{A}=۶۰ \qquad a=BC=۲۰ \qquad b=AC=\cancel{۲۰}\sqrt{۶}/\cancel{۲}$

$\hat{B}+\hat{C}=۱۲۰^\circ \Rightarrow ۱۸۰^\circ-۱۲۰^\circ=۶۰^\circ \qquad \dfrac{۲۰}{\sin ۶۰^\circ}=\dfrac{۲۰}{\sqrt{۳}/۲}=\dfrac{۴۰}{\sqrt{۳}} \Rightarrow \dfrac{۲۰\sqrt{۶}/۲}{\sin\hat{B}}=\dfrac{۴۰}{\sqrt{۳}}$

$\Rightarrow \dfrac{a}{\sin\hat{A}}=\dfrac{b}{\sin\hat{B}}=\dfrac{c}{\sin\hat{C}}$

$\Rightarrow \sin\hat{B}=\dfrac{۱۰\sqrt{۶}\times\sqrt{۳}}{۴\sqrt{۳}\times ۱۰ ۲}=\dfrac{\sqrt{۲}}{۲} \Rightarrow \hat{B}=۴۵^\circ \Rightarrow \hat{C}=۱۸۰-(\underbrace{۴۵^\circ+۶۰^\circ}_{۱۰۵})=۷۵^\circ \;\big|\; ۱۲۰-۴۵=۷۵^\circ$

۲-

$۱۸۰-(\underbrace{۶۰+۷۵}_{۱۳۵})=۴۵^\circ \Rightarrow \dfrac{a}{\sin\hat{A}_1}=\dfrac{۲\sqrt{۲}}{\sqrt{۲}/۲}=۴ \Rightarrow \dfrac{AC}{\sin ۶۰^\circ}=۴ \Rightarrow AC=۲\sqrt{۳}$

$\dfrac{AC}{\sin\hat{B}}=\dfrac{۲\sqrt{۳}}{۱}=\dfrac{x}{\sin\hat{A}} \Rightarrow a=۲\sqrt{۳}\times\dfrac{\sqrt{۳}}{۲}=۳$

$\sin\hat{B} \to ۹۰^\circ \qquad \sin\hat{A} \to ۶۰^\circ \qquad \sin ۶۰^\circ \to \sqrt{۳}/۲$

۳-

$\dfrac{c}{\sin\hat{C}}=\dfrac{b}{\sin\hat{B}} \Rightarrow \dfrac{\sin\hat{C}}{\sin\hat{B}}=\dfrac{c}{b} \Rightarrow (b^۲-۲)\left(\dfrac{c}{b}\right)=c \Rightarrow b^۲-۲=b \Rightarrow b^۲-b-۲=۰$

$\Rightarrow (b-۲)(b+۱)=۰ \Rightarrow b=۲$

۴-

$b^۲=a^۲+c^۲-۲ac\cos\hat{B} \Rightarrow ۱۳^۲=۲۱^۲+۲۰^۲-۲\times۲۱\times۲۰\times\cos\hat{B}$

$b \to ۱۳ \quad a \to ۲۱ \quad c \to ۲۰$

$\Rightarrow ۱۶۹=\underbrace{۴۴۱+۴۰۰}_{۸۴۱}-۸۴۰\cos\hat{B} \Rightarrow -۶۷۲=-۸۴۰\cos\hat{B} \Rightarrow \cos\hat{B}=\dfrac{۶۷۲}{۸۴۰}=۰/۸$

$۶۷۲ \to ۳۳۶ \to ۱۶۸ \qquad ۸۴۰ \to ۴۲۰ \to ۲۱۰$

$\Rightarrow \sin\hat{B}=۰/۶ \longrightarrow ۱-\dfrac{۱۶}{۲۵}=\dfrac{۹}{۲۵}$

۵- الف)

$$a^2 = b^2 + c^2 - \cancel{2}bc\cos\hat{A}$$

$$b^2 \to 16,\quad c^2 \to 4(1+\sqrt{3})^2,\quad \cos\hat{A} \to \tfrac{1}{\cancel{2}}$$

$$\Rightarrow a^2 = 16 + \underbrace{4(1+\sqrt{3})^2}_{4(4+2\sqrt{3})} - \underbrace{8(1+\sqrt{3})}_{-8-8\sqrt{3}} = 16+16+\cancel{8\sqrt{3}}-8-\cancel{8\sqrt{3}}$$

$$\Rightarrow a^2 = 24 \Rightarrow a = \sqrt{24} = 2\sqrt{6}$$

ب)

$$\frac{a}{\sin\hat{A}} = \frac{b}{\sin\hat{B}} = \frac{c}{\sin\hat{C}}$$

$$\Rightarrow \frac{\sqrt{24}}{\frac{\sqrt{3}}{2}} = 2\sqrt{6}\times\frac{2}{\sqrt{3}} = 4\sqrt{2} = \frac{4}{\sin\hat{B}} \Rightarrow \sin\hat{B} = \frac{4}{4\sqrt{2}} \Rightarrow \hat{B} = 45^\circ$$

$$C = 180 - (\underbrace{45^\circ + 60^\circ}_{105}) = 75^\circ$$

-6

$BC = AC \Rightarrow \hat{\alpha} = \hat{B} \Rightarrow \hat{C} = 180 - (2\alpha)$ $\quad \frac{c}{\sin\hat{C}} = \frac{BC}{\sin\hat{\alpha}} \Rightarrow \frac{1}{\sin\hat{C}} = \frac{2}{\sin\hat{\alpha}}$

$\Rightarrow 2\sin\hat{C} = \sin\alpha \Rightarrow 2\sin(180-2\alpha) = \sin\alpha \Rightarrow 2\sin 2\alpha = 2(2\cancel{\sin\alpha}\cos\alpha) = \cancel{\sin\alpha}$

$\Rightarrow \cos\alpha = \frac{1}{4}$ $\quad \Rightarrow DE = \sqrt{36+16-48\times\frac{1}{4}} = \sqrt{52-12} = \sqrt{40} = 2\sqrt{10}$

$a^2 = b^2 + c^2 - 2\cancel{b}c\cos\hat{A}$

-7 الف)

$a^2 = b^2 + c^2 - 2bc\cos\theta$ $\quad BC = \sqrt{64+25-80\times\frac{1}{2}} = \sqrt{49} = 7$

ب)

$S_{\widehat{BAC}} = 8\times 5\times\sin 60\times\frac{1}{2} = \frac{20\sqrt{3}}{2} = 10\sqrt{3}$

-8

$(b+c)^2 = b^2+c^2+2bc$

$a^2 = b^2+c^2-2bc\cos\hat{A}$

$\Rightarrow \frac{\cancel{b^2}+\cancel{c^2}+2bc-\cancel{b^2}-\cancel{c^2}+2bc\cos\hat{A}}{bc(\cos\hat{A}+1)} = \frac{2bc(\cos\hat{A}+1)}{bc(\cos\hat{A}+1)} = 2$

$\cancel{a^{۳}}+b^{۳}-c^{۳}=\cancel{a^{۳}}+ba^{۲}-ca^{۲} \Rightarrow (b-\cancel{c})(b^{۲}+c^{۲}+bc)=a^{۲}(b-\cancel{c}) \Rightarrow a^{۲}=b^{۲}+c^{۲}+bc$ -۹

$a^{۲}=b^{۲}+c^{۲}-۲bc\cos\hat{A} \Rightarrow \cancel{b^{۲}}+\cancel{c^{۲}}-۲bc\cos\hat{A}=\cancel{b^{۲}}+\cancel{c^{۲}}+bc \Rightarrow -۲b\cancel{c}\cos\hat{A}=b\cancel{c}$

$\Rightarrow \cos\hat{A}=-\cancel{۱}/\cancel{۲}$

$S_{ABC}=(۳-\cos\theta)(۱+۳\cos\theta)\cancel{\sin ۳۰}\times\cancel{۱/۲}=۴$ $\qquad \Rightarrow ۳+۹\cos\theta-\cos\theta-۳\cos^{۲}\theta=۸$ -۱۰

$\downarrow$ $۱/۲$

$\Rightarrow ۳\cos^{۲}\theta-۸\cos\theta+۵=۰ \longrightarrow \cos\theta=+۱$

$\longrightarrow \cos\theta=+\cancel{c/a}=+۵/۳\ \times$

$\Rightarrow a^{۲}=b^{۲}+c^{۲}-۲bc\cos\cancel{\hat{A}}$ $\quad ۳۰$

$\Rightarrow a^{۲}=۴+۱۶-\cancel{۱۶}^{\,۸}\times\sqrt{۳}/\cancel{۲}$

$\Rightarrow a^{۲}=۲۰-۸\sqrt{۳}$