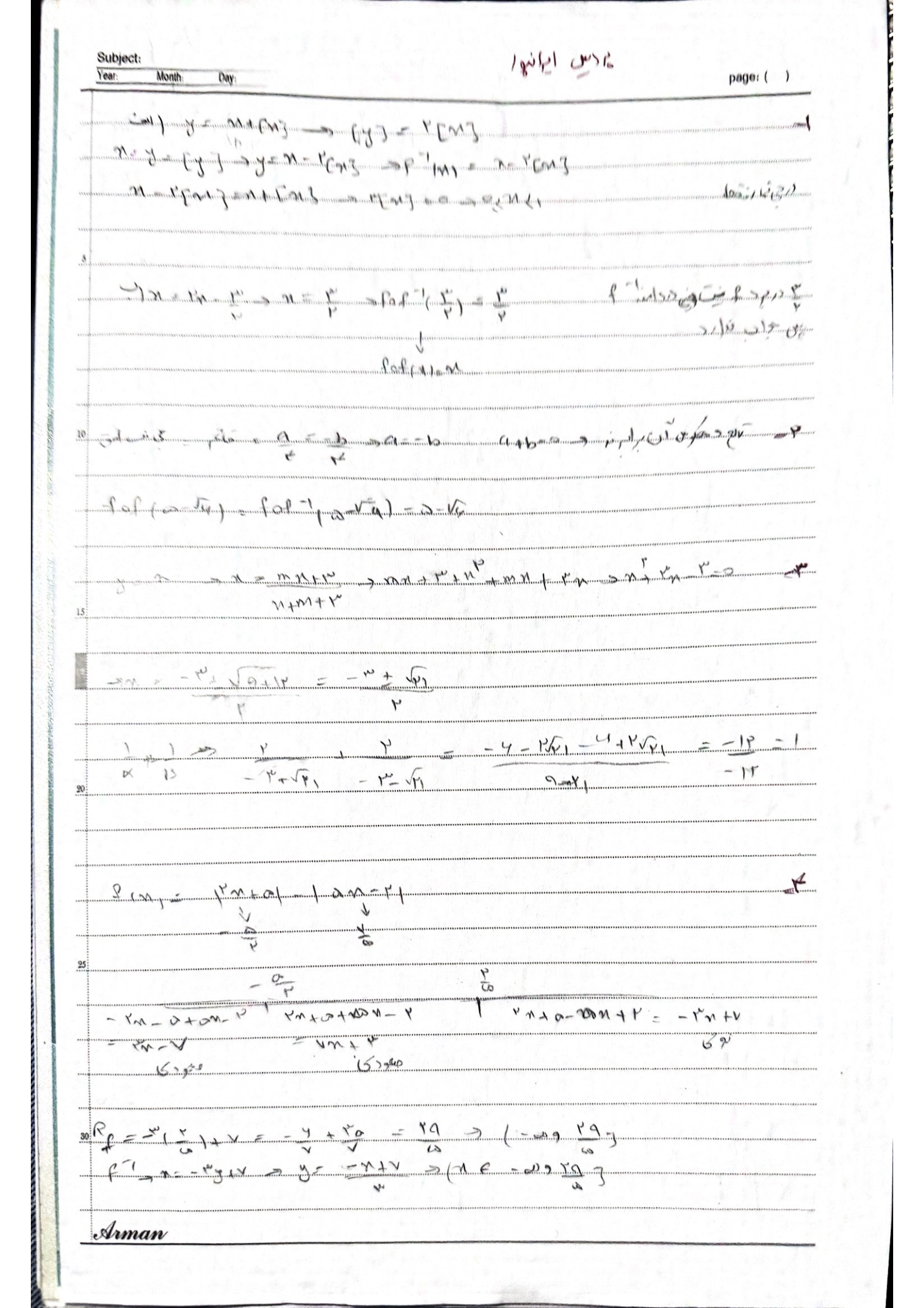
جلسه اول



۵ـ

$g(x) = y \Rightarrow g^{-1}(y) = x$

$y = 2f(3x) - 1 \Rightarrow f(3x) = \frac{y+1}{2} \Rightarrow 3x = f^{-1}\left(\frac{y+1}{2}\right) \Rightarrow x = \frac{1}{3} f^{-1}\left(\frac{y+1}{2}\right)$

$g^{-1}(y) = x \Rightarrow g^{-1}(y) = \frac{1}{3} f^{-1}\left(\frac{y+1}{2}\right) \Rightarrow a = \frac{1}{3} \quad b = 1 \quad c = 2 \quad d = 0$

$\frac{ac+d}{2b} = \frac{\frac{2}{3}}{2} = \frac{1}{3}$

۶ـ

$f(x) = x + 2\sqrt{x} \Rightarrow x \geq 0 \Rightarrow R_f = 0 + 2\sqrt{0} = 0 \Rightarrow R_f = [0, +\infty)$

$x = y + 2\sqrt{y} + 1 - 1 \Rightarrow x = (\sqrt{y}+1)^2 - 1 \Rightarrow x + 1 = (\sqrt{y}+1)^2$

$\sqrt{x+1} = \sqrt{y} + 1 \Rightarrow \sqrt{y} = \sqrt{x+1} - 1 \Rightarrow y = x + 1 + 1 - 2\sqrt{x+1}$

$\Rightarrow y = x + 2 - 2\sqrt{x+1} \Rightarrow x \geq 0$

۷ـ در نقطه تقاطع نیمساز

$x = y \Rightarrow x = x - 1 + 2^x - x \Rightarrow 1 = 2^x \Rightarrow x = 0$

۸ـ

$f^{-1}(x) = t \Rightarrow f(t) = x = t^3 - 4t$

$x f^{-1}(x) - x \geq 0 \Rightarrow t - (t^3 - 4t) \geq 0 \Rightarrow t^3 + 3t \leq 0$

$\Rightarrow t(t^2+3) \leq 0$ $\quad - \ \overset{0}{|} \ + \quad$ $t = (-\infty, 0]$ اعداد صحیح منفی

۹ـ

$f^{-1}(x+4) = x - 3 \Rightarrow f(x-3) = x + 4 \Rightarrow f(t) = t + 7$

$-t + \sqrt{t+4} = t + 7 \Rightarrow (2t+7)^2 = t + 4 \Rightarrow 4t^2 + 27t + 45 = 0$

$t = \frac{-27 \pm \sqrt{27^2 - 16 \times 45}}{8} \Rightarrow \boxed{t = -3} \ , \ -3.75$

ق بخورد

۱۰ـ

$\sqrt{y} = 2 - \sqrt{x} \Rightarrow y = (2 - \sqrt{x})^2 \Rightarrow fof(x) = \left(2 - \sqrt{(2-\sqrt{x})^2}\right)^2$

$fof(x) = (2 - |2 - \sqrt{x}|)^2 \Rightarrow (2 - 2 + \sqrt{x})^2 = |x|$

$x \geq 0$

$2 - \sqrt{x} \geq 0 \Rightarrow \sqrt{x} \leq 2 \Rightarrow 0 \leq x \leq 4$