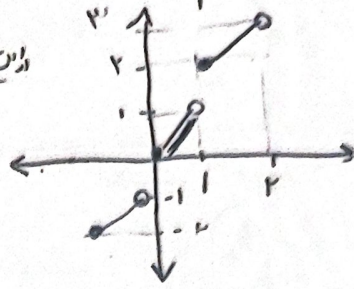


تکلیف عدد 11

طالع دارهم 2 عدد اضطرار

$$f(x) = x + [x]$$

$$y = x + \frac{[y]}{2}$$



$$f^{-1}(x) = ? \rightarrow [x + [x]] = [y] \rightarrow [y] = 2[x]$$

$$\hookrightarrow [x] = \frac{[y]}{2}$$

این بازه ها را در جابجایی
مقادیر اند

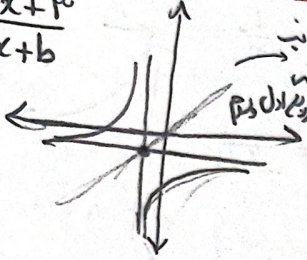
$$x = y \cdot \frac{[y]}{2} \rightarrow f^{-1}(x) = x \cdot \frac{[x]}{2}$$

$$f^{-1}(x) = f(x) \rightarrow x - \frac{[x]}{2} = [x] + x \rightarrow \frac{[x]}{2} = 0 \rightarrow x = 0$$

$$f \circ f^{-1}(x) = 2x - \frac{[x]}{2} \rightarrow x = 2x - \frac{[x]}{2} \rightarrow \frac{[x]}{2} = x$$

(ب)

$$y = \frac{ax+b}{cx+d}$$



این تابع و وارزش برده
مستقیم هستند
اگر $a+b=0 \rightarrow a=-b$
و اگر $c+d=0$ است
این تابع وارزش برده
مستقیم هستند

$$f \circ f^{-1}(x) = ? \rightarrow f \circ f^{-1}(x) = x$$

این تابع وارزش برده
مستقیم هستند

$$f(x) = \frac{mx+n}{x+m+n}$$

$$f^{-1}(x) = \frac{(-m-n)x+n}{x-m}$$

$$\frac{mx+n}{x+m+n} = \frac{-m-n}{x-m}$$

$$-m = m+n \rightarrow -n = 2m \rightarrow m = -\frac{n}{2}$$

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$$f(x) = \sqrt{(2x+5)^2} - \sqrt{(5x-2)^2} \rightarrow |2x+5| - |5x-2|$$

$$y = \begin{cases} -3x+5 & x > \frac{2}{5} \\ 7x+5 & -\frac{2}{5} \leq x \leq \frac{2}{5} \\ 3x-5 & x \leq -\frac{2}{5} \end{cases} \rightarrow y = -3x+5 \rightarrow \frac{y-5}{-3} = x \rightarrow y = -\frac{1}{3}x + \frac{5}{3}$$

4

$$g(x) = 2f(2x) - 1$$

$$g^{-1}(x) = 2f^{-1}\left(\frac{x+1}{2}\right) + 1$$

5

$$f(x) = x + 2\sqrt{x} \rightarrow f^{-1}(x) = \sqrt{x+1} - 1$$

$$\hookrightarrow (\sqrt{x+1})^2 - 1 \hookrightarrow D_f = [0, +\infty) \quad D_{f^{-1}} = [0, +\infty)$$

$$R_f = [0, +\infty) \quad R_{f^{-1}} = [0, +\infty)$$

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$$y = \frac{x-1}{2} + 2^x \rightarrow 2^x + x - 1 = x \rightarrow 2^x = 1 \rightarrow x = 0$$

7

4

$$f(x) = x^3 + 5x \rightarrow \text{monotonically increasing}$$

$$y = \sqrt{f^{-1}(x) - x} \rightarrow 0 = f^{-1}(x) - x \rightarrow f(f^{-1}(x)) = f(x) \rightarrow x > f(x)$$

$$\rightarrow x > x^3 + 5x \rightarrow x^3 + 5x < 0 \rightarrow x(x^2 + 5) < 0 \rightarrow x < 0$$

→ x < 0

$$f(x) = -x + \sqrt{x+5}, x > -5 \rightarrow$$

4

$$\sqrt{x} + \sqrt{y} = 1 \rightarrow \sqrt{y} = 1 - \sqrt{x} \rightarrow y = (1 - \sqrt{x})^2$$

$$f \circ f(x) = ? \rightarrow (1 - (1 - \sqrt{x})^2)^2 = (1 - 1 + 2\sqrt{x} - x)^2 = (2\sqrt{x} - x)^2 = 4x - 4x\sqrt{x} + x^2$$

$$\rightarrow \boxed{x^2 + 4x\sqrt{x} + 4x} \rightarrow (x + 2\sqrt{x})^2$$

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