

نام و نام خانوادگی: پاسخنامه تشریحی تکلیف شماره کلاس: A	
الف) $f(x) = x + [x]$ $\rightarrow f(x) = f^{-1}(x)$ $\rightarrow f \circ f^{-1}(x) = x - \frac{x}{2} \rightarrow x - \frac{x}{2} = x$ $y = x + [x] \rightarrow x = y + [y]$ $x + [x] = x - \frac{[x]}{2}$ $f \circ f^{-1}(x) = x$ $[x] = [y + [y]] \rightarrow [x] = [y]$ $\frac{x}{2} [x] = 0$ $x \in D_{f^{-1}}$ $x \in R_f$ $[y] = \frac{[x]}{2} \rightarrow x = y + \frac{[x]}{2}$ $\frac{x}{2} [x] = 0$ $x \in R_f$ $y = x - \frac{[x]}{2} \rightarrow f^{-1}(x) = x - \frac{[x]}{2}$ $\checkmark$	۱
$y = \frac{ax+b}{cx+d}$ $\rightarrow cx+b=0 \rightarrow x = -\frac{b}{c}$ $x=y \rightarrow \frac{b}{x} = \frac{a}{x} \rightarrow a=-b$ $\rightarrow f \circ f^{-1}(x) = f \circ f^{-1}(x) = x \rightarrow f \circ f^{-1}(x) = x$ $\checkmark$	۲
$f(x) = \frac{mx+r}{x+(m+r)}$ $\rightarrow \frac{mx+r}{x+(m+r)} = \frac{-(m+r)x+r}{x-m} \rightarrow mx^2 - m^2x + rx - rm = -(m+r)x - (m+r)x + r(m+r)$ $\rightarrow \frac{1}{x} + \frac{1}{x} = \frac{a+b}{ab} \cdot \frac{c}{p} = \frac{-b}{a} = \frac{-b}{c} = \frac{-(4m+9)}{-(4m+9)} = 1 \cdot (m + \frac{9}{4})$	۳
$f(x) = \sqrt{(x+d)^2} - \sqrt{(dx-r)^2} = x+d - dx-r \rightarrow x \geq \frac{r}{d} \rightarrow x+d-dx+r = -rx+v$ $x \geq \frac{r}{d} \rightarrow -rx \leq \frac{-r}{d} \rightarrow -rx+v \leq \frac{-r}{d}$ $y = -rx+v \rightarrow v-y = rx \rightarrow x = \frac{v-y}{r}$ $y = \frac{v-x}{r}$ $\rightarrow f^{-1}(x) = \frac{v-x}{r}$ $x \leq \frac{19}{2}$ $\checkmark$ $D_f = R_{f^{-1}} = [\frac{r}{d}, +\infty)$ $R_f = D_{f^{-1}} = [-\infty, \frac{19}{2}]$	۴
$g(x) = f(f(x)) - 1 = t \Rightarrow g(x) = t \Rightarrow x = g^{-1}(t)$ $f(f(x)) = t+1 \Rightarrow f(x) = \frac{t+1}{r} \Rightarrow rx = f^{-1}(\frac{t+1}{r})$ $\Rightarrow g^{-1}(t) = \frac{f^{-1}(\frac{t+1}{r})}{r} \Rightarrow g^{-1}(x) = \frac{1}{r} f^{-1}(\frac{x+1}{r}) + \frac{b}{c} \Rightarrow \frac{ac+d}{rb} = \frac{\frac{1}{r} \times r + 0}{r \times 1} = \frac{1}{r}$ $\checkmark$	۵

$$f(x) = x + \sqrt{x} = (\sqrt{x+1})^2 - 1 \rightarrow (\sqrt{x+1})^2 = y+1 \Rightarrow \sqrt{x+1} = \sqrt{y+1} \Rightarrow \sqrt{x} = \sqrt{y+1} - 1$$

min
 $\sqrt{x+1} = 1 \Rightarrow x=0 \rightarrow y \geq 0$

$$\rightarrow x = (\sqrt{y+1} - 1)^2 \rightarrow y = (\sqrt{x+1} - 1)^2 \rightarrow f^{-1}(x) = (\sqrt{x+1} - 1)^2 \checkmark$$

$$Df = Rf = [0, +\infty) \checkmark$$

$$Rf = Df = [0, +\infty)$$

$$y = x - 1 + \sqrt{x} \rightarrow \text{معمولی است} \rightarrow \text{با دو C جدا از هم نقطه (x, y) پیدا می کند} \rightarrow x = x - 1 + \sqrt{x}$$

$$1 = \sqrt{x}$$

$$x = 0$$

$$y = 0 \rightarrow \text{معمولی است} \checkmark$$

$$f(x) = x^2 + 5x \rightarrow \text{معمولی است}$$

$$y = \sqrt{f^{-1}(x)} - x \xrightarrow{\text{معمولی است}} f^{-1}(x) \geq x \rightarrow x, f(x) \rightarrow x, x^2 + 5x \rightarrow x^2 + 5x \leq 0$$

$$x(x+5) \leq 0 \Rightarrow x \leq 0$$

معمولی است

$$f(x) = -x + \sqrt{x+5} \xrightarrow{\text{معمولی است}} x = -y + \sqrt{y+5} \xrightarrow{\text{معمولی است}} x = -(-y) + \sqrt{-y+5}$$

$$x = -(y+5) + \sqrt{y+5} \rightarrow y+5 = - (y+5) + \sqrt{y+5} \rightarrow t-1 = -t+\sqrt{t} \rightarrow 2t-1 = \sqrt{t}$$

$$y = x - 5 \rightarrow x = y + 5$$

$$\hookrightarrow t = 4t^2 - 4t + 1 \rightarrow 4t^2 - 4t + 1 = 0 \rightarrow t = \frac{4 \pm \sqrt{16-4}}{8} \rightarrow t = \frac{4 \pm \sqrt{12}}{8} \rightarrow t = \frac{4 \pm 2\sqrt{3}}{8} \rightarrow t = \frac{2 \pm \sqrt{3}}{4}$$

$y+5=1 \rightarrow y=-5 \Rightarrow x=0$ معمولی است

$y+5=\frac{1}{4} \rightarrow y=-\frac{19}{4} \Rightarrow y > -5 \Rightarrow \text{معمولی است}$

$$\sqrt{x} + \sqrt{y} = 2 \xrightarrow{\text{معمولی است}} \sqrt{y} + \sqrt{x} = 2 \rightarrow f(x) = f^{-1}(x)$$

$$f \circ f(x) = f \circ f^{-1}(x) = x \Rightarrow$$

$$Df = Rf \Rightarrow x=0 \rightarrow y=5 \Rightarrow Df = Rf = [0, 5] \checkmark$$

