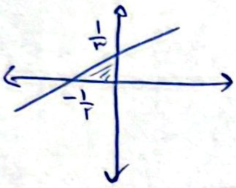
تکالیف عطار — فاطمه شعبانی

1) $2y = 3x - 1 \to$

f^{-1}: $2x = 3y - 1 \to 3y = 2x + 1 \to y = \frac{2}{3}x + \frac{1}{3} \to \frac{2}{3}x + \frac{1}{3} = 0 \to \frac{2}{3}x = -\frac{1}{3} \to x = -\frac{1}{2}$



$\to S = \dfrac{\frac{1}{3}\times\frac{1}{2}}{2} = \boxed{\frac{1}{12}}$

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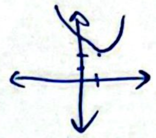
2) الف) $y = x^2 - 2x + 3$; $x \in [1, +\infty) \to y = \underbrace{x^2 - 2x + 1}_{\text{مربع کامل}} + 2$



یک به یک ✓

$\to y = (x-1)^2 + 2 \to y - 2 = (x-1)^2 \to \pm\sqrt{y-2} = x - 1 \xrightarrow[\text{عکس}]{\text{جای x و y}} y = \pm\sqrt{x-2} + 1$

$\to \boxed{f^{-1} = \sqrt{x-2} + 1 \; ; \; x \in [2, +\infty)}$

قابل قبول با توجه به اینکه دامنه‌ی تابع f = برد f^{-1} و برد $[1, +\infty)$ است!

ب) $y = x^2 - 2x + 3$; $x \in [3, +\infty)$

$R_{f^{-1}} = [3, +\infty) \to \sqrt{x-2} + 1 \geq 3 \to \sqrt{x-2} \geq 2$; $f^{-1} = y = \sqrt{x-2} + 1$ ← با توجه به قسمت الف)

$x - 2 \geq 4 \to x \geq 6$ ← $R_{f^{-1}} = D_f$ که می‌دانیم

$\to \boxed{f^{-1} = \sqrt{x-2} + 1 \; ; \; x \in [6, +\infty)}$

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3) $f(x) = 2x - \sqrt{16 - 4x(4-x)} \to$

$f(x) = 2x - \sqrt{\underbrace{16 - 16x + 4x^2}} \to 2x - \sqrt{(2x-4)^2} = 2x - |2x-4|$

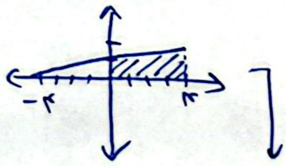
$4x^2 - 16x + 16 = (2x-4)^2$

$\to y = 2x - |2x-4| \to x \geq 2 : 2x - 2x + 4 = 4 \to$ وارون ناپذیر ××

↳ $x < 2 : 2x + 2x - 4 = 4x - 4$ ✓

دامنه f = برد f^{-1} *

$\to y = 4x - 4 \to y + 4 = 4x \xrightarrow{\text{جای x و y عوض}} \underline{\frac{1}{4}x + 1 = y} \; f^{-1}$



* ↳ $\frac{1}{4}x + 1 < 2 \to \frac{1}{4}x < 1 \to \underline{x < 4}$

$\dfrac{(1+2)4}{2} = \boxed{6}$ ← S با ناحیه مثبت x

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4) $y = \dfrac{x}{\sqrt{x^2 - 5}} \to \begin{matrix} y_1 = y_2 \\ x_1 = x_2 \end{matrix} \to \dfrac{x_1^2}{\sqrt{x_1^2 - 5}} = \dfrac{x_2^2}{\sqrt{x_2^2 - 5}} \xrightarrow{\text{با توجه به دامنه}} \dfrac{x_1^2}{x_1^2 - 5} = \dfrac{x_2^2}{x_2^2 - 5} \to$

x و y هم علامت‌اند!

$\to \cancel{x_1^2 x_2^2} - 5x_1^2 = \cancel{x_2^2 x_1^2} - 5x_2^2 \to x_1^2 = x_2^2 \to x_1 = \pm x_2 \to x_1 = x_2$ ✓

با توجه به مورد x و y هم علامت بودن

یک به یک ✓

معکوس سازی کن

$\to y = \dfrac{x}{\sqrt{x^2 - 5}} \to y^2 = \dfrac{x^2}{x^2 - 5} \to x^2 = \dfrac{5y^2}{y^2 - 1} \to x = \pm\sqrt{\dfrac{5y^2}{y^2 - 1}}$

هم علامت بودن x و y

x و y عوض

$\to$ $\boxed{f^{-1} \to y = \pm \dfrac{|x|\sqrt{5}}{\sqrt{x^2 - 1}}}$

5) $g(x) = \begin{cases} x^2 - 4x + k & ; x \geq 2 \\ -x^2 + 6x + 3k & ; x < 2 \end{cases} \to -\frac{b}{2a} = \frac{4}{2} = 2$

$\hookrightarrow -\frac{b}{2a} = \frac{-6}{-2} = 3$

1) $\xrightarrow{x=2} 4 - 8 + k = k - 4$ min

2) $\xrightarrow{x=2} -4 + 12 + 3k = 8 + 3k$ max

$\to 8 + 3k < k - 4 \to 2k < -12 \to \boxed{k < -6}$

6) $3x + |x| = y \to 4x \; ; x \geq 0 \quad y = 4x \to x = \frac{y}{4} \quad x \geq 0$

$\quad \hookrightarrow 2x \; ; x \leq 0 \to y = 2x \to x = \frac{y}{2} \quad x \leq 0$

$\to f^{-1} = \begin{cases} \frac{x}{4} & x \geq 0 \\ \frac{x}{2} & x \leq 0 \end{cases} \to ax + b|x| \to x(a+b) \; ; x \geq 0 \;\; \hookrightarrow x(a-b) \; ; x \leq 0 \to \begin{cases} a+b = \frac{1}{4} \\ a-b = \frac{1}{2} \end{cases} \to a = \frac{3}{8} \quad b = \frac{-1}{8}$

$\to ab = \frac{3}{8} \times \frac{-1}{8} = \boxed{\frac{-3}{64}}$

7) از نقطه $(2, -10)$ عکس $\to \frac{3(2)+4}{2+m} = -10 \to \frac{10}{2+m} = -10 \to \underline{m = -3}$

$\to f \circ f^{-1}(x) = x \to x + \frac{3x+4}{x-3} = \frac{x^2 - 3x + 3x + 4}{x-3} = x \to \frac{x^2+4}{x-3} = x \to$

$f^{-1}(x) = \frac{3x+4}{x-3}$ طبق روش $\frac{ax+b}{cx+d}$ — مخرج از فاکتور ۳

$\cancel{x^2} - 3x = \cancel{x^2} + 4$

$-3x = 4 \to \boxed{x = \frac{-4}{3}}$

8) $f(3-2x) = 2 - g(\frac{x}{2}) \to 4g^{-1}(4) + f^{-1}(-2) = ? \to -2 = 2 - (4) \to$

$\begin{cases} f(3-2x) = -2 \\ g(\frac{x}{2}) = 4 \end{cases}$ در معکوس عکس $\to 4(\frac{x}{2}) + 3 - 2x$

$= \cancel{2x} + 3 - \cancel{2x} = \boxed{3}$

9) $y = \frac{5x-3}{1-x} \xrightarrow[\text{مبدا}]{\text{قرینه}} -x = \frac{-5y-13}{1+y} \to x = \frac{5y+13}{1+y} \to y = \frac{-x+13}{1-5x} \xrightarrow{(x-2)} y = \frac{-(x-2)+13}{1-5(x-2)}$

$\to y = \frac{-x+15}{-5x-9} \xrightarrow{y-3} y = \frac{x-15}{5x+9} - 3 = \frac{x - 15 - 15x - 27}{5x+9} = \frac{-14x-42}{5x+9} = y$

$f^{-1} = \frac{-9x-42}{5x+14} \to \frac{-9x+42}{5x+14} = \frac{-14x-42}{5x+9} \to -70x^2 - 406x + 588 = -45x^2 + 129x + 378$

$\frac{ax+b}{cx+d} \to f^{-1} = f$

$\to -25x - 435x + 210 = 0 \to$

$\sum = \frac{-b}{a} = \boxed{\frac{435}{-25}}$

10) $y = x + 4[x] \to x = y + 4[y]$

$\to [y] = [x + 4[x]] \to [y] = [x] + 4[x] = 5[x] \to [x] = \frac{[y]}{5} \to [y] = \frac{[x]}{5}$

$\to [y] = \frac{[x]}{5} \to x - 4[y] = y \to x - \frac{4[x]}{5} = y \to g(x) = x - \frac{4[x]}{5} \quad f(x) = x + 4[x]$

$x = y + 4[y]$

$(f-g)(x) = x + 4[x] - x + \frac{4[x]}{5} = \boxed{\frac{24}{5}[x]}$