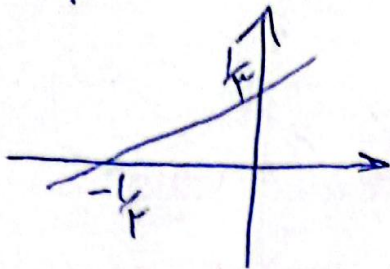


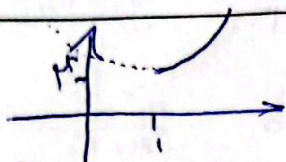
$$y = \frac{3x-1}{y} \rightarrow x = \frac{3y-1}{y} \rightarrow y^{-1} = \frac{3x+1}{3}$$



مساحت = $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27}$ ✓

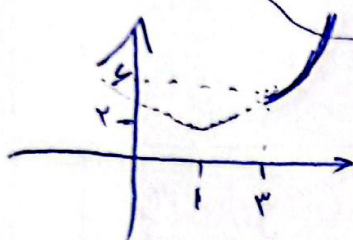
(۲)

$$\frac{-b}{2a} = 1$$



الف) تابع یکتا است. $y = (x-1)^2 + 2$ معکوس: $x = (y-1)^2 + 2$

ب) یک به یک است. $y = (x-1)^2 + 2$ معکوس: $x = (y-1)^2 + 2$



دومین: $y^{-1} = \sqrt{x-2} + 1$ $D y^{-1} = [2, +\infty)$ ✓

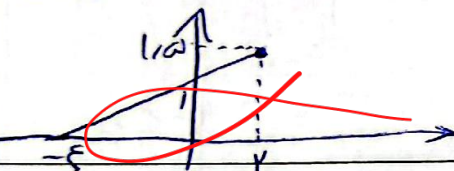
(۲)

$$f(x) = 2x - \sqrt{16 - 4x(x-2)} = 2x - 2\sqrt{(x-1)^2} = 2x - 2|x-1|$$

۱) $x \geq 1 \rightarrow 2x - 2x + 2 = 2$ وارون پذیر

$Df^{-1} = Rf = (-\infty, 2]$

۲) $x < 1 \rightarrow 2x - 2(1-x) = 4x - 2$ وارون پذیر: $x = \frac{y+2}{4} \rightarrow y^{-1} = \frac{x+2}{4}$ ✓



۳) $\frac{(1+1/a)x^2}{2} = \frac{1}{2}$ ✓

۴) $y_1 = y_2 \rightarrow x_1 = x_2$ $\frac{x_1}{\sqrt{x_1^2 - a}} = \frac{x_2}{\sqrt{x_2^2 - a}}$ $\frac{x_1^2}{x_1^2 - a} = \frac{x_2^2}{x_2^2 - a} \rightarrow x_1^2 x_2^2 - a x_1^2 = x_1^2 x_2^2 - a x_2^2 \rightarrow x_1 = \pm x_2$

۵) $x = \frac{y}{\sqrt{y^2 - a}} \rightarrow x^2 = \frac{y^2}{y^2 - a} \rightarrow x^2 y^2 - a x^2 = y^2 \rightarrow y^2 = \frac{a x^2}{x^2 - 1}$ ✓

۶) $g(x) = \begin{cases} (x-2)^2 - 4 + k & x \geq 2 \\ -(x-3)^2 + 9 + 3k & x < 2 \end{cases}$

$\Rightarrow x=2 \rightarrow -4+k \geq -1+9+3k \rightarrow k \leq -12$ ✓

$\Rightarrow k \leq -6$ ✓

(۲)

$$f(x) = 3x + |x| \xrightarrow{x \geq 0} \varepsilon x = y \text{ معكوس: } y^{-1} = \frac{x}{\varepsilon}$$

$$\xrightarrow{x < 0} y x = y \text{ معكوس: } y^{-1} = \frac{x}{\varepsilon}$$

$$g(x) = a x + b |x| \xrightarrow{x \geq 0} (a+b)x = \frac{x}{\varepsilon} \rightarrow a+b = \frac{1}{\varepsilon} \Rightarrow 2a = \frac{1}{\varepsilon}$$

$$\xrightarrow{x < 0} (a-b)x = \frac{x}{\varepsilon} \rightarrow a-b = \frac{1}{\varepsilon}$$

$$\rightarrow a = \frac{1}{2\varepsilon} \rightarrow b = -\frac{1}{2\varepsilon} \quad ab = \frac{1}{\varepsilon} \times -\frac{1}{\varepsilon} = -\frac{1}{\varepsilon^2} \quad \checkmark$$

$$f(2) = -10 \rightarrow \frac{\varepsilon + \delta}{\varepsilon + m} = -10 \rightarrow m = -3 \rightarrow f(x) = \frac{3x + \varepsilon}{x - 3}$$

$$a+d=0 \rightarrow f=f^{-1} \rightarrow f^{-1}(x) = \frac{3x + \varepsilon}{x - 3} \quad y = f \circ f^{-1}(x) + f^{-1}(x)$$

$$= x + \frac{3x + \varepsilon}{x - 3} = \frac{x^2 + \varepsilon}{x - 3} \quad \text{معكوس: } \frac{x^2 + \varepsilon}{x - 3} = x \rightarrow$$

$$x^2 - 3x = x^2 + \varepsilon \rightarrow 3x + \varepsilon = 0 \rightarrow x = -\frac{\varepsilon}{3} \quad \checkmark$$

$$f(3-2x) = 2 - g(g(x)) = y \rightarrow 3-2x = f^{-1}(y) \text{ و } g^{-1}(2-y) = \frac{x}{\varepsilon}$$

$$\rightarrow \frac{3}{\varepsilon} - \frac{1}{\varepsilon} f^{-1}(y) = 2g^{-1}(2-y) \quad y = -2 \quad \frac{3}{\varepsilon} - \frac{1}{\varepsilon} f^{-1}(y) = 2g^{-1}(2-y)$$

$$\frac{3}{\varepsilon} - \frac{1}{\varepsilon} f^{-1}(-2) = 2g^{-1}(\varepsilon) \xrightarrow{\times \varepsilon} 3 - f^{-1}(-2) = \varepsilon g^{-1}(\varepsilon) \rightarrow$$

$$\varepsilon g^{-1}(\varepsilon) + f^{-1}(-2) = 3 - f^{-1}(-2) + f^{-1}(-2) = 3 \quad \checkmark$$

$$y = \frac{\varepsilon x - 13}{1-x} \xrightarrow{\text{معكوس}} \frac{\varepsilon x + 13}{x+1} \xrightarrow{\text{معكوس}} \frac{\varepsilon(x-2) + 13}{x-2+1} - 3 = \frac{2x + \varepsilon}{x-1} = f(x)$$

$$\text{معكوس: } x = \frac{y + 2y}{-1+y} \rightarrow y = \frac{x + \varepsilon}{x-2} = f^{-1}(x)$$

$$\text{معكوس: } f^{-1}(x) = f(x) \rightarrow \frac{x + \varepsilon}{x-2} = \frac{2x + \varepsilon}{x-1} \rightarrow x^2 - 3x - \varepsilon = 0$$

$$\rightarrow x_1 = \frac{3 + \sqrt{9 + 4\varepsilon}}{2} \Rightarrow x_1 + x_2 = \frac{3 + \sqrt{9 + 4\varepsilon}}{2} + \frac{3 - \sqrt{9 + 4\varepsilon}}{2} = \frac{6}{2} = 3 \quad \checkmark$$

$$y = x + \varepsilon[x] \text{ معكوس: } x = y + \varepsilon[y] \rightarrow y = x - \varepsilon[y]$$

$$[y]: y = x - \varepsilon[y] \xrightarrow{\text{معكوس}} [y] = [x - \varepsilon[y]] \rightarrow [y] = \frac{[x]}{\varepsilon}$$

$$y = x - \varepsilon[y] \rightarrow y = x - \frac{\varepsilon}{\varepsilon} [x] = g(x)$$

$$(f-g)(x) = x + \varepsilon[x] - x + \frac{\varepsilon}{\varepsilon} [x] = \frac{\varepsilon}{\varepsilon} [x] \quad \checkmark$$