

الف) $y^3 - 2x^2 = 2x - 1 \rightarrow y^3 = 2x^2 + 2x - 1 \rightarrow y = \sqrt[3]{2x^2 + 2x - 1}$ تابع است.

ب) $x \cos y = y \cos x \xrightarrow{x=\frac{\pi}{2}} \frac{\pi}{2} \cos y = y \cos \frac{\pi}{2} \rightarrow \cos y = 0$  $x = \frac{\pi}{2} \rightarrow y = k\pi + \frac{\pi}{2}$
 $x = \frac{3\pi}{2} \rightarrow y = k\pi - \frac{\pi}{2}$ تابع نیست.

ج) $\frac{x+y}{2} = \sqrt{xy} \rightarrow x+y = 2\sqrt{xy} \rightarrow (\sqrt{x} - \sqrt{y})^2 = 0 \rightarrow \sqrt{x} = \sqrt{y} \rightarrow x = y$ تابع است.

د) $xy^2 + 2x - y^3 - 2y = 0 \rightarrow x(y^2 + 2) - y(y^2 + 2) = 0 \rightarrow (y^2 + 2)(x - y) = 0 \rightarrow x - y = 0$
 $x = y$ تابع است.

$f(x) = \begin{cases} 2x^2 - b & |x-1| \geq 2 \\ a + 2x & -2 \leq x \leq -1 \end{cases}$ $|x-1| \geq 2 \rightarrow \begin{cases} x-1 \geq 2 \rightarrow x \geq 3 \\ x-1 \leq -2 \rightarrow x \leq -1 \end{cases}$ $(-\infty, -1] \cup [3, +\infty)$

$x = -1 \rightarrow a - 2 = 2 - b \rightarrow a + b = 6$

در هر دو معادله صدق می کند

$f(x) = \sqrt{-x^2 - bx + a}$ $b = -1$
 $a = 2 \rightarrow a + b = 1$

$x = a \rightarrow -a^2 - ab + a = 0 \rightarrow a(a - b + 1) = 0 \rightarrow a \neq 0 \rightarrow a = 1 - b$

$x = b \rightarrow -b^2 - b^2 + a = 0 \rightarrow -2b^2 = -a \rightarrow 2b^2 = 1 - b \rightarrow 2b^2 + b - 1 = 0$
 $b = -1 \checkmark$
 $b = \frac{1}{2}$ غلط
 $b \in \mathbb{Z}$

$y = \frac{2}{\sqrt{ax^2 + bx + c}}$ $D_f = (-\infty, 2)$ $\frac{2}{\frac{1}{0} -} \rightarrow$ ریشه مضرب $\rightarrow$ غیر منافع $\Rightarrow a = 0$

$ax^2 + bx + c \rightarrow a = 0 \rightarrow bx + c > 0$ $b < 0 \rightarrow x < -\frac{c}{b}$

$2b + c = 0 \rightarrow b = -\frac{c}{2}$

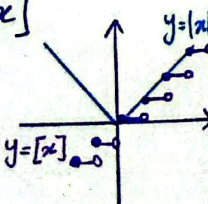
$3a + c - 3|b| \rightarrow -2b - 3(-b) = b$ $c = -2b$

الف) $f(x) = \sqrt[3]{\frac{1}{|x| - [x]}}$
 $\rightarrow |x| - [x] \neq 0 \rightarrow |x| \neq [x]$

$D_f = \mathbb{R} - (\mathbb{Z} \cup \{0\})$

ب) $f(x) = \sqrt{x} + \sqrt{y} = 9$
 $I \Rightarrow x, y \geq 0$
 $\sqrt{y} = 9 - \sqrt{x} \rightarrow 9 - \sqrt{x} \geq 0 \rightarrow 9 \geq \sqrt{x} \rightarrow x \leq 81$
 $II \rightarrow x \leq 1$

$D_f = [0, 81]$



$$\text{الف) } f(x) = \sqrt{\log^3 x - 1}$$

$$\text{I} \Rightarrow 3x - 1 > 0 \rightarrow x > \frac{1}{3}$$

$$\text{II} \Rightarrow x > 0, x \neq 1$$

$$\text{III} \Rightarrow \log^3 x - 1 \geq 0 \rightarrow 3x - 1 \geq 1$$

$$D_f = \left[\frac{1}{3}, +\infty\right) - \{1\}$$

$$\text{ب) } f(x) = \log \frac{1}{4 + \sqrt{|x|} - |x|}$$

$$4 + \sqrt{|x|} - |x| > 0$$

$$\sqrt{|x|} = t \rightarrow -(t^2 - t - 4) > 0 \rightarrow t^2 - t - 4 < 0 \rightarrow \frac{1 \pm \sqrt{17}}{2}$$

$$-2 < \sqrt{|x|} < 3 \rightarrow \sqrt{|x|} < 3 \rightarrow |x| < 9 \rightarrow -9 < x < 9$$

$$D_f = (-9, 9)$$

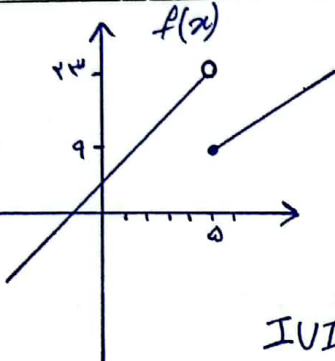
$$f(x) = \sqrt{\frac{3x+2}{x+b}} + a \rightarrow \frac{3x+2+a(x+b)}{x+b} \geq 0 \quad D_f = (2, +\infty) \rightarrow \frac{2}{-1} \rightarrow -2$$

$$b = -2$$

$$f(x) = \begin{cases} 2x-1 & [x] > 4 \\ 5x+3 & [x] \leq 4 \end{cases}$$

$$[x] > 4 \rightarrow x > 4$$

$$[x] \leq 4 \rightarrow x \leq 4$$



$$y = \sqrt{f(x) - x + 1} \rightarrow f(x) - x + 1 \geq 0 \rightarrow f(x) \geq x - 1$$

$$\text{I} \Rightarrow 2x - 1 \geq x - 1 \rightarrow x \geq 0$$

$$\text{II} \Rightarrow 5x + 3 \geq x - 1 \rightarrow x \geq -\frac{4}{4} = -1$$

$$\text{I} \cup \text{II} \Rightarrow D_f = [-1, +\infty)$$

$$y = \frac{2x^3 + 34x^2 + 11x + 3}{(x^2 + 2x + 1)^2} \rightarrow \frac{3(2x+1)^3}{14(x+\frac{1}{2})^2} = \frac{3(2x+1)^3}{1(2x+1)^2}$$

$$\rightarrow \frac{3}{1(2x+1)} \rightarrow D_f = \mathbb{R} - \left\{-\frac{1}{2}\right\}$$

$$y = \sqrt{1 - \log(2x-a)}$$

$$\text{I} \Rightarrow 2x - a > 0 \rightarrow 2x > a \rightarrow x > \frac{a}{2}$$

$$\text{II} \Rightarrow 1 - \log^2 x - a \geq 0 \rightarrow \log^2 x - a \leq 1$$

$$\frac{a}{2} < x \leq \frac{a+10}{2}$$

$$2x - a \leq 10 \rightarrow 2x \leq a + 10 \rightarrow x \leq \frac{a+10}{2}$$

$$D_f = (b, c] \rightarrow b = \frac{a}{2} \rightarrow c - b = \frac{a+10}{2} - \frac{a}{2} = \frac{10}{2} = 5$$