

تلف

A دوازدهم رختان

الف) $2 \sin^2 \left(\alpha + \frac{\pi}{4} \right) - 1$

$\sin^2 \alpha \rightarrow 0$

$\rightarrow 1$

$y = -1$

$Ry = [-1, 1]$

①

ب) $2 \cos^2 \alpha - 1$

$\cos^2 \alpha \rightarrow -1$

$\rightarrow 1$

$y = -1$

$Ry = [-1, 1]$

$y = \frac{\alpha}{|\alpha|} + [\alpha]$

$\alpha \neq 0$

$\alpha \in [-3, 3]$

②

$-3 < \alpha < -2$

$[\alpha] = -3$

$y = \frac{\alpha}{-\alpha} - 3 = -3$

$-2 < \alpha < -1$

$[\alpha] = -2$

$y = \frac{\alpha}{-\alpha} - 2 = -2$

$-1 < \alpha < 0$

$[\alpha] = -1$

$y = \frac{\alpha}{-\alpha} - 1 = -1$

$0 < \alpha < 1$

$[\alpha] = 0$

$y = \frac{\alpha}{\alpha} + 0 = 1$

$1 < \alpha < 2$

$[\alpha] = 1$

$y = \frac{\alpha}{\alpha} + 1 = 2$

$2 < \alpha < 3$

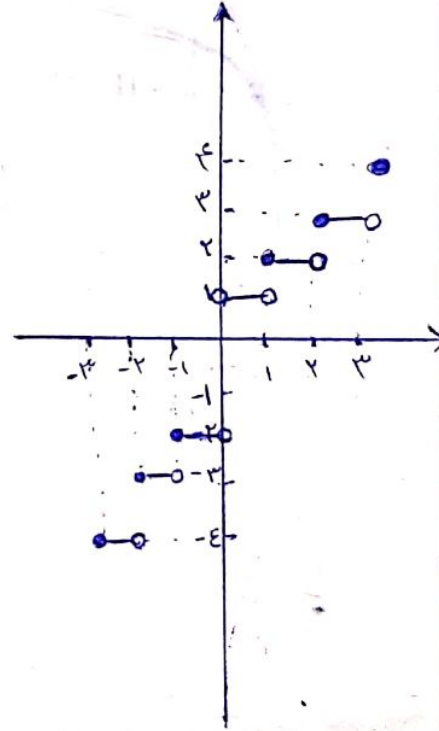
$[\alpha] = 2$

$y = \frac{\alpha}{\alpha} + 2 = 3$

$\alpha = 3$

$[\alpha] = 3$

$y = \frac{\alpha}{\alpha} + 3 = 4$

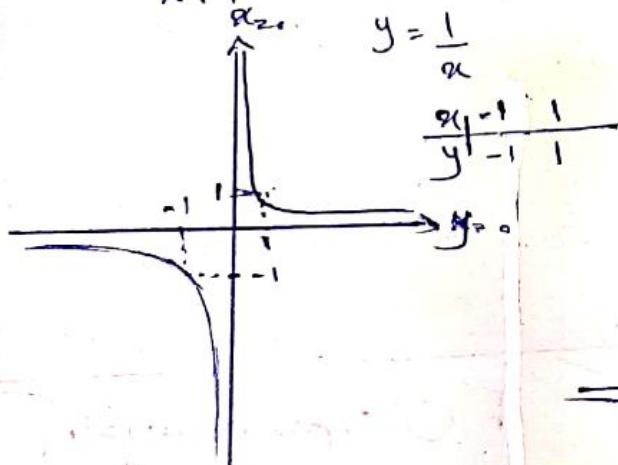


$$y = \left| \frac{5x+1}{x+3} \right|$$

$$= 5 - \frac{11}{x+3}$$

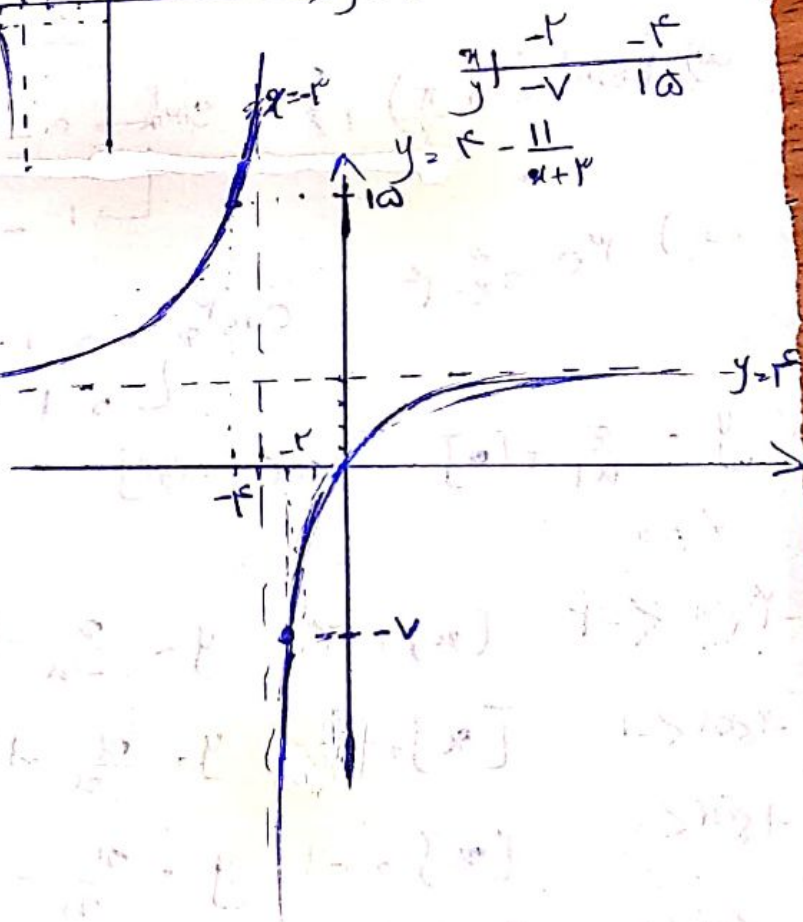
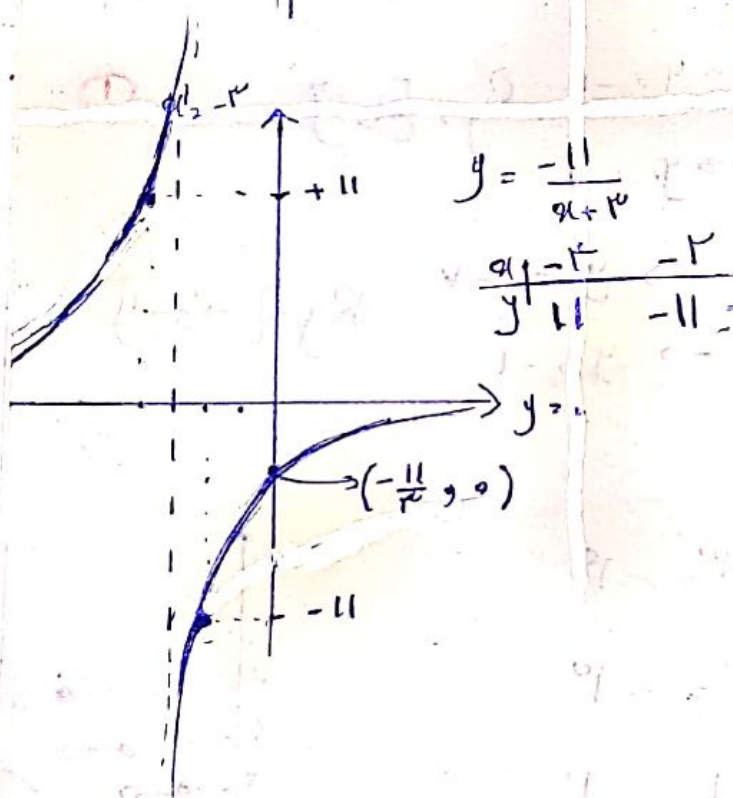
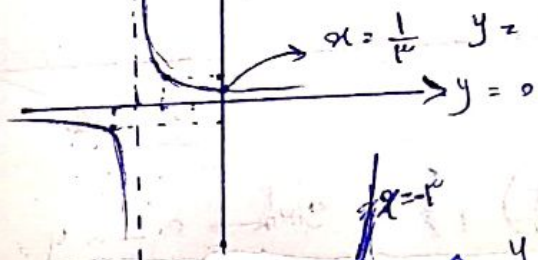
$$y' = \frac{5x+1}{x+3} = \frac{5x+1+11-11}{x+3} = \frac{5(x+3)}{x+3} - \frac{11}{x+3}$$

$x+3=0 \rightarrow x=-3 \leftarrow$ جانب افقی



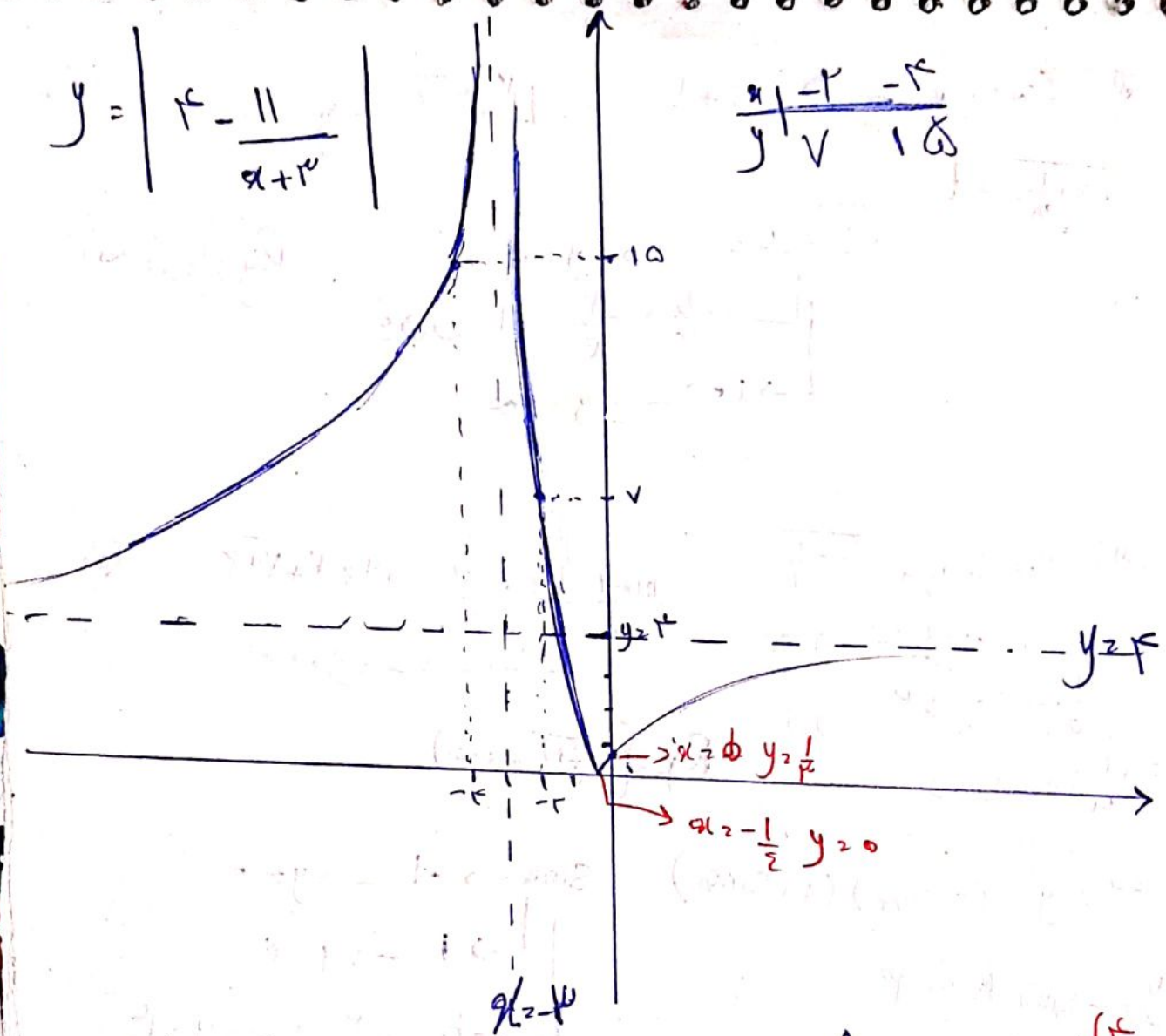
جانب ایست $y = \frac{a}{c} = +3$

$$y = \frac{1}{x+3} \quad \begin{array}{c|c} x & -4 \\ \hline y & -1 \end{array}$$



$$y = \left| \frac{x-1}{x+1} \right|$$

$$\frac{x-1}{y} = \frac{-1}{1 \pm y}$$



$$|x-1| + |y-1| = 1$$

$$|y-1| = 1 - |x-1|$$

$$x \geq 1 \quad 1 - (x-1) = \pm(y-1)$$

$$1) \quad y \geq 1 \quad 1 - x + 1 = y - 1$$

$$\boxed{-x + y = y}$$

$$2) \quad y < 1 \quad 1 - x + 1 = 1 - y$$

$$\boxed{x - 1 = y}$$

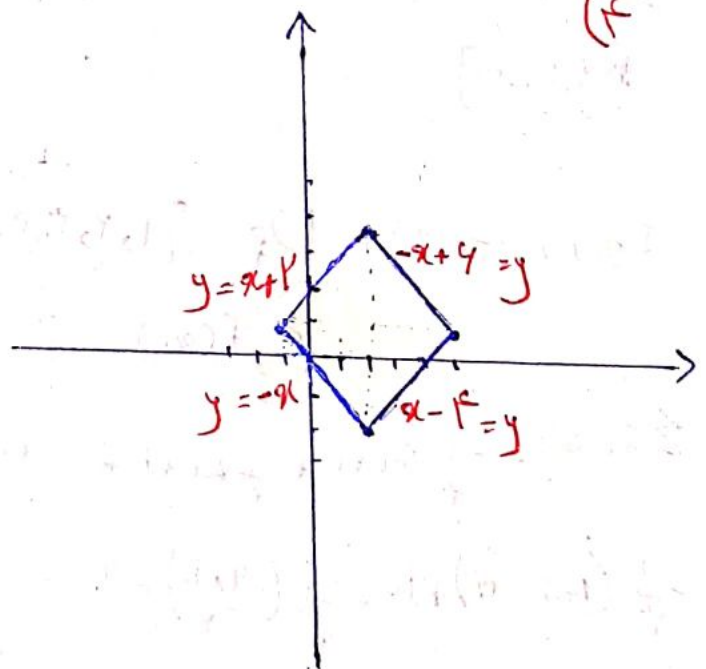
$$x < 1 \quad 1 - (1-x) = \pm(y-1)$$

$$1) \quad 1 - 1 + x = y - 1 \quad \boxed{x + 1 = y}$$

$$y \geq 1$$

$$2) \quad 1 - 1 + x = 1 - y \quad \boxed{-x = y}$$

$$y < 1$$



الف) $f(x) = x + \frac{1}{x} + \sqrt{x + \frac{1}{x}}$

$x + \frac{1}{x} \begin{cases} \xrightarrow{a < 0} a + \frac{1}{a} < -2 \text{ غلط} \\ \xrightarrow{a > 0} a + \frac{1}{a} \geq 2 \text{ صحيح} \end{cases}$

$x + \frac{1}{x} > 0$ چون زیر اصفال

$x + \frac{1}{x} \begin{cases} \min \rightarrow 2 \rightarrow y = 2 + 2\sqrt{2} \\ \max \rightarrow +\infty \rightarrow y = +\infty \end{cases}$

(د)

$R_f = [2 + 2\sqrt{2}, +\infty)$

ب) $y = (x - \sin x)(1 + \sin x)$

$y = -\sin^2 x + \sin x + x$

$R_y = [0, \pi]$

$\sin x \rightarrow -1 \rightarrow y = 0$

$\rightarrow 1 \rightarrow y = \pi$

$\rightarrow -\frac{b}{2a} = -\frac{\pi}{-2} = \frac{\pi}{2} \rightarrow y = \pi$

$f(x) = -\frac{1}{x^2} x + 1$ $D_f = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

(4)

$\sum f(x_i) = ?$

$\sum f(x_i) = -\frac{1}{x^2} x_1 + 1 + -\frac{1}{x^2} x_2 + 1 + \dots + -\frac{1}{x^2} x_9 + 1 = -\frac{1}{x^2} (x_1 + x_2 + \dots + x_9) + (9 \times 1)$

$-\frac{1}{x^2} (1 + 2 + \dots + 9) + 9 = -\frac{1}{x^2} \left(\frac{9 \times 10}{2} \right) + 9 = -\frac{45}{x^2} + 9 = \frac{36 - 45}{x^2} = \frac{-9}{x^2}$

$$y = \sqrt{ax^2 + bx + c} \quad D_f = [r, +\infty) \quad \frac{y}{-x} + \quad (V)$$

$$y = \sqrt{bx + c} \quad x=r \rightarrow \sqrt{rb + c} = 1 \quad rb + c = 1$$

$$y = \sqrt{bx + c} \quad x=r \rightarrow \sqrt{rb + c} = 0 \quad \begin{cases} rb + c = 1 \\ rb + c = 0 \end{cases}$$

$$g(x) = \sqrt{x^2 + 0 - 1x(-1)} = \sqrt{x^2 + x} \quad b=1 \rightarrow c=-1 \quad y' = x^2 + x \quad R_y = [r, +\infty)$$

$$R_g = [r, +\infty) \rightarrow [r, +\infty)$$

$$f(x) = \frac{x^2 - 1}{x+1} \quad D_f = \mathbb{R} - \{a, b\} \quad R_f = [c, +\infty) - \{1\} \quad (A)$$

$$f(x) = \frac{x(x-1)}{x+1} = \frac{x(x-1)(x+1)}{(x+1)} = x^2 - 1 \quad x \rightarrow r \rightarrow r^2 - 1 = 1 \quad r^2 - 1 = 1$$

$$x^2 - 1 = 1 \quad x^2 - 1 - 1 = 0 \quad (x-1)(x+1) = 0 \rightarrow x \neq 1$$

$$D_f = \mathbb{R} - \{1, -1\} \quad a=r \quad b=-1 \quad y = x^2 - 1 \quad as = -(-1) = 1$$

$$R_f = [-1, +\infty) - \{1\} \quad c=-1 \quad y_s = -1 \quad R_y = [-1, +\infty)$$

$$f\left(\frac{a+b}{rc}\right) = f\left(\frac{1-1}{-1}\right) = f(-1) = \frac{(-1)^2 - 1(-1)}{-1+1} = \frac{-1+1}{1} = 0$$

$$y = \sqrt[3]{x^3 + ax^2 + bx - 1} \rightarrow$$

$$y = \sqrt[3]{(x+a)^3 - 1} \quad a = -2$$

$$y = \sqrt[3]{(x-2)^3} = \sqrt[3]{x^3 + 12x - 4x^2 - 1}$$

$$D_f = [-4, 12]$$

$$y = x - 2$$

$$R_f = [f(-4), f(12)] = [-1, 10]$$

(9)
لعل ضابطه زیر را کامل کرد
و جواب کامل باشد

$$y = [x] + [x + \frac{1}{x}] \quad \frac{1}{x} \leq x < 2$$

$$\frac{1}{x} \leq x < 1 \quad [x] = 0 \quad [x + \frac{1}{x}] = 1 \quad y = 1$$

$$1 \leq x < 1.5 \quad [x] = 1 \quad [x + \frac{1}{x}] = 1 \quad y = 2$$

$$1.5 \leq x < 2 \quad [x] = 1 \quad [x + \frac{1}{x}] = 2 \quad y = 3$$

