

① الف) $f(x) = \sqrt{x} \ln \left(x + \frac{\pi}{x}\right) - 1$

$\max_{x \in \mathbb{R}^+} f(x) = 1 \Rightarrow R_f = [-1, 1]$

$\min_{x \in \mathbb{R}^+} f(x) = -1$

ب) $f(x) = \sqrt{x} \ln x - 1$

$\max_{x \in \mathbb{R}^+} f(x) = -1$

$\min_{x \in \mathbb{R}^+} f(x) = -v \Rightarrow R_f = [-v, -1]$

② $-x \leq x < x \rightarrow \frac{x}{|x|} = -1, [x] = -x \Rightarrow y = -x$

$-x \leq x < -1 \rightarrow \frac{x}{|x|} = -1, [x] = -x \rightarrow y = -x$

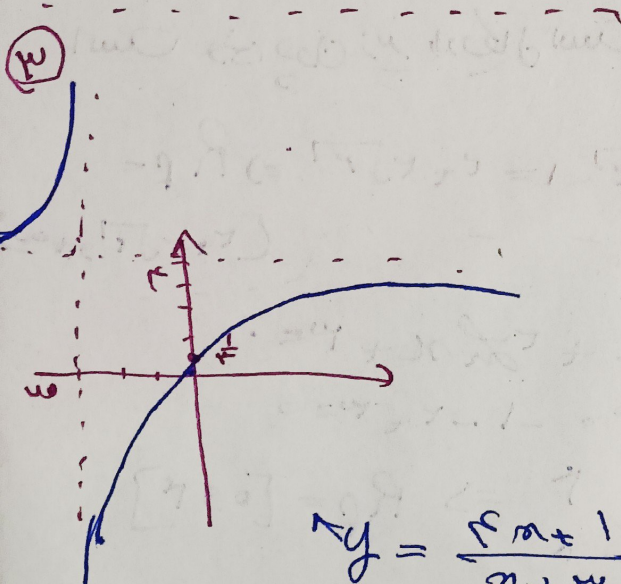
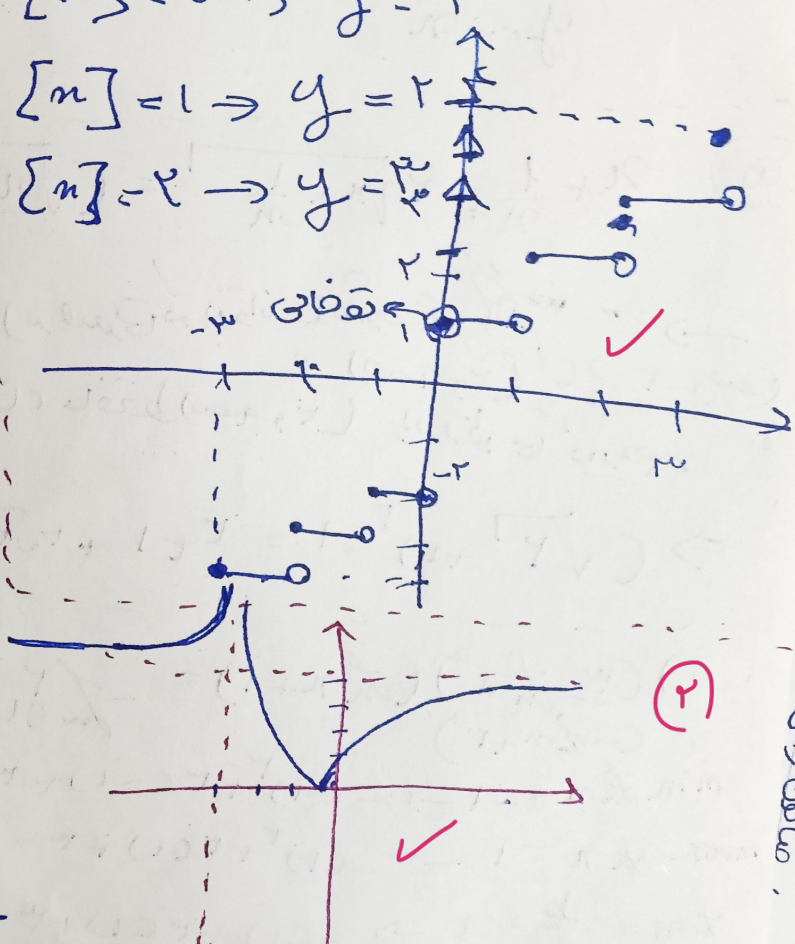
$-1 \leq x \leq 0 \rightarrow \frac{x}{|x|} = -1, [x] = -1 \rightarrow y = -1$

$0 \leq x < 1 \rightarrow \frac{x}{|x|} = 1, [x] = 0 \rightarrow y = 1$

$1 \leq x < 2 \rightarrow \frac{x}{|x|} = 1, [x] = 1 \rightarrow y = 2$

$2 \leq x < 3 \rightarrow \frac{x}{|x|} = 1, [x] = 2 \rightarrow y = 3$

$x = x \rightarrow y = x$

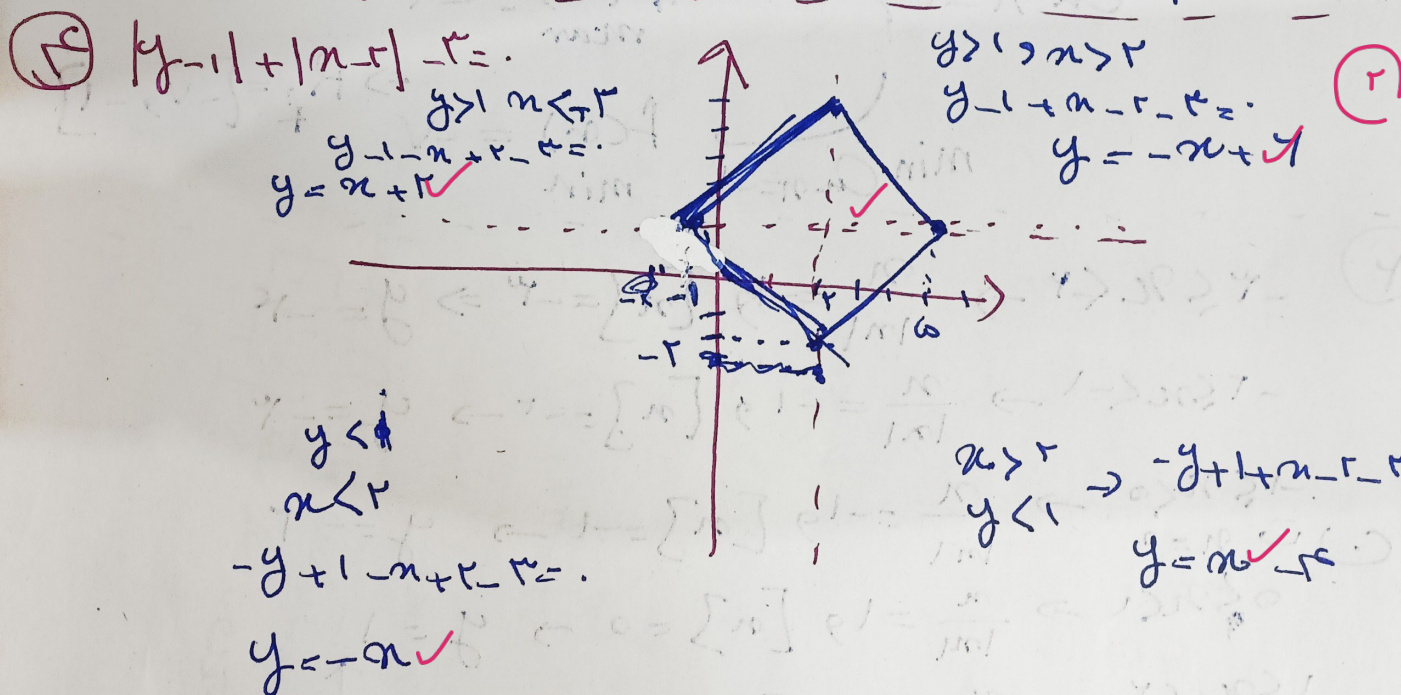


$y = \frac{x+1}{x+2}$

$\lim_{n \rightarrow \infty} \frac{(n-r)(n+r)}{n(n+r)} = \lim_{n \rightarrow \infty} \frac{n-r}{n} = 1$

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$(2) n + \frac{1}{n} + r\sqrt{n + \frac{1}{n}} = (\sqrt{n + \frac{1}{n}} + 1)^2 - 1$

این تابع را می‌توانیم به صورت زیر بنویسیم: $f(x) = x + \frac{1}{x} + r\sqrt{x + \frac{1}{x}}$

این تابع برای $x > 0$ تعریف شده است.

$\Rightarrow (\sqrt{r} + 1)^2 - 1 = r + 1 + r\sqrt{r} - 1 = r + r\sqrt{r} \Rightarrow R_p = [r + r\sqrt{r}, +\infty)$

$(\frac{r}{n} \ln n) (\ln n + 1) = -\ln n + r \ln n + r = 0$

$\min \ln n = -1 \rightarrow -(-1)^r + r(-1) + r = -1 - r + r = 0$

$\max \ln n = 1 \rightarrow -(1)^r + r(1) + r = -1 + r + r = 2r - 1 \Rightarrow R_p = [0, 2r - 1]$

$\ln n = -\frac{b}{a} = 1 \rightarrow -(1) + r(1) + r = r$

(9) $D_f = \{1, 2, \dots, 9\} \rightarrow (x) =$ (2)

$$\Rightarrow 9 \times 10 + \left\{ -\frac{1}{x} \left(\underbrace{1 + \frac{1}{x} + \frac{1}{x^2}}_{\frac{9 \times 10}{x} = 90} \right) \right\} = 90 + \left(-\frac{90}{x} \right) = 0$$

(V) $D_f = [r_0 + \infty)$ ————— $\Rightarrow a = 0$ ، $r_0 =$ (1, 1.5)

$\Rightarrow r_0 b + c = 0$
 $r_0 b + c = 1 \Rightarrow b = 1, c = -1$

$g(x) = \sqrt{x^2 + 1} \Rightarrow R_g = [r_0 + \infty)$ کامل بند نیست

(A) $\frac{a(n-2)(n+2)}{n+2} \Rightarrow A(n) = n^2 - 4n$ (2)

$2^2 - 4n = 1 \Rightarrow n^2 - 4n - 1 = 0$
 $n = \frac{4 \pm \sqrt{16 + 4}}{2} = \frac{4 \pm \sqrt{20}}{2}$

$R_f = [-1, +\infty) \rightarrow c = 1$

$A\left(\frac{5-1}{2}\right) = A(-1) = 3$

(a) $\sqrt[3]{(a-1)^3} = n-1$

$= n^3 - 4n^2 + 13n - 1 \rightarrow a = 9, b = 12$ (1, 5)

تابع قطعی از زیر رادیکال نگیرد

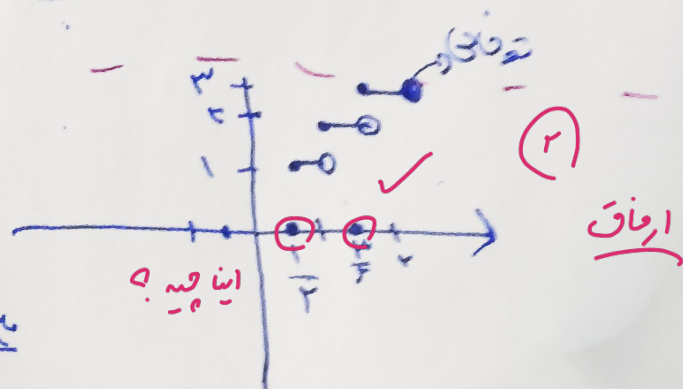
$D[-9, 1) \Rightarrow R_f = [1, 9)$

(10) $y = \{n\} + \left\{ n \cdot \frac{1}{x} \right\}$

$\frac{1}{x} \leq n < 1 \rightarrow y = 1$

$1 < n < \frac{1}{x} \rightarrow y = 2$

$\frac{1}{x} \leq n < 2 \rightarrow y = 3$



۷- با توجه به دامنه، زیر رادیکال خط با شیب مثبت بوده یعنی $a=0$ است و نقطه $(2,0)$ هم در معادله خط صدق می‌کند...

$$\begin{cases} (2,0) \rightarrow 2b+c=0 \\ (3,1) \rightarrow 3b+c=1 \end{cases} \rightarrow b=1, c=-2$$

$$g(x) = \sqrt{x^2+4} \rightarrow \text{ext} \Big|_x^0 \Rightarrow R_g = \sqrt{[4, +\infty)} = [2, +\infty)$$

۹- چون عدد ثابت عبارت زیر رادیکال، -8 است پس حتماً $(x-2)^3$ بوده و داریم:

$$y = \sqrt[3]{x^3 - 4x^2 + 12x - 8} \rightarrow a = -4, b = 12 \rightarrow D_y = [-4, 12]$$

$$x = -4 \rightarrow y = -8$$

$$x = 12 \rightarrow y = 10$$

$$\rightarrow R_y = [-8, 10]$$