

۱) بردارهای

الف) $f(x) = 2 \sin^2(x + \frac{\pi}{6}) - 1$

$2 \sin^2(x + \frac{\pi}{6}) \leq 1$

$-1 \leq 2 \sin^2(x + \frac{\pi}{6}) \leq 1 \Rightarrow R_f = [-1, 1]$

$-1 \leq 2 \sin^2(x + \frac{\pi}{6}) - 1 \leq 1$

ب) $g(x) = 3 \cos^3 x - 4$

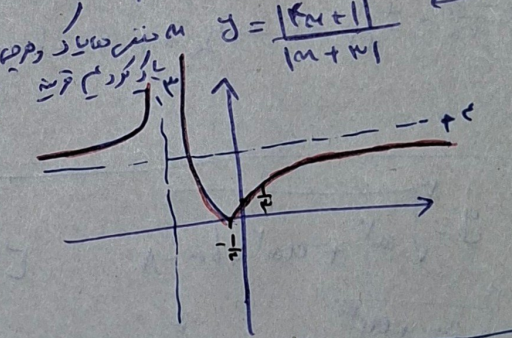
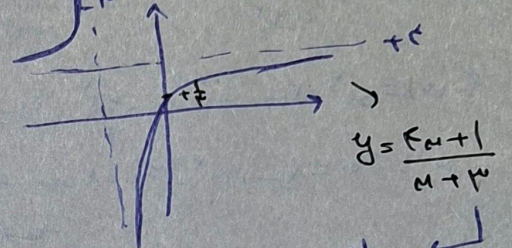
$-1 \leq \cos^2 x \leq 1$

$-3 \leq 3 \cos^3 x \leq 3 \Rightarrow R_g = [-3, 3]$

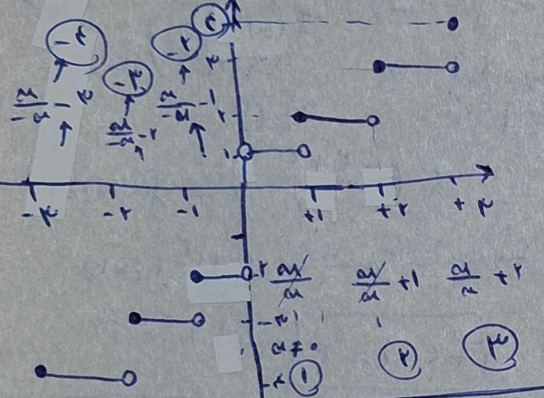
$-4 \leq 3 \cos^3 x - 4 \leq -1$

۲) $y = \frac{|x+1|}{|x+3|}$

$x \rightarrow -3^- \rightarrow y \rightarrow +\infty$
 $x \rightarrow -3^+ \rightarrow y \rightarrow -\infty$
 $x = 0 \rightarrow y = \frac{1}{3}$



$y = \frac{x}{|x|} + [x]$ $R_f = [-1, 1]$

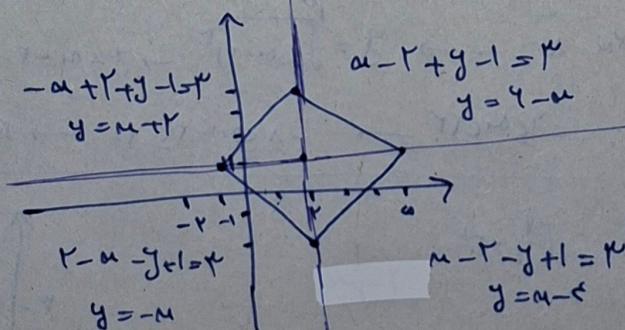


$f(x+1) = 0 \leftarrow 0 = |f(x)|$
 $f(x) = -1$
 $x = -\frac{1}{2}$

$y = \frac{|x+1|}{|x+3|}$

$|x-2| + |y-1| = 3$
 $x=2$ $y=1$

$-x+2+y-1=3$
 $y=x+2$



$x-2-y+1=3$
 $y=x-4$

الف) $f(x) = x + \frac{1}{x} + 2 \sqrt{x + \frac{1}{x}}$

$f(x) = (\sqrt{x + \frac{1}{x}})^2 + 2 \sqrt{x + \frac{1}{x}}$

$\rightarrow R_f = [2 + 2\sqrt{2}, +\infty)$

ب) $y = (3 - \sin x)(1 + \sin x)$

$\sin x \rightarrow 1$
 $\sin x \rightarrow 0$
 $\sin x \rightarrow -1$
 $1 = \frac{b}{a}$

$R_f = [0, 4]$

تجزیه اعداد صحیح

$f(x) = -\frac{1}{x}x + 10$ $R_f = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$(-\frac{1}{x} \times 1 + 10) + (-\frac{1}{x} \times 2 + 10) + (-\frac{1}{x} \times 3 + 10) + \dots + (-\frac{1}{x} \times 10 + 10)$

$-\frac{1}{x}(1+2+\dots+10) + 10 \times 10 = -\frac{55}{x} + 100 = 0$

$\downarrow$
 $\frac{55}{x} = 100$

$f(u) = \sqrt{au^r + bu + c}$ $D_f = [r, +\infty)$ $f(r) = 1$ $g(u) = \sqrt{bu^r + au - r}$ $D_g = [r, +\infty)$

if $\Delta \geq 0 \rightarrow f(r)$ if $\Delta \geq 0 \rightarrow g(r) = -R$
 $\Rightarrow a = 0 \rightarrow f(u) = \sqrt{bu + c}$
 $f(r) = 1 \rightarrow 1 = \sqrt{rb + c}$ $0 = \sqrt{rb + c}$
 $\begin{cases} rb + c = 0 \\ rb + c = 1 \end{cases}$
 $b = 1 \rightarrow c = -r$
 $g(u) = \sqrt{au^r + r}$
 $a_3 = -\frac{b}{r} = 0$
 $g_5 = r$
 $R_g = [r, +\infty) = [r, +\infty)$

$f(u) = \frac{u^r - r}{u + r} = \frac{u(u-r)(u+r)}{u+r} = u(u-r)$ $R = \{a, b\} \rightarrow \dots$
 $f(u) = u^r - r$ $a = -r$ $(-r)^r - r(-r) = 1$
 $u^r - r = 1 \rightarrow u^r - r = 1 \rightarrow (u-r)(u+r) = 1$
 $u \neq r$ $u \neq -r$
 $a_3 = -\frac{b}{r} = \frac{+r}{+r} = 1 \rightarrow g_5 = 1 - r = -1 \rightarrow R = [-1, +\infty) - \{1\}$
 $f(-\frac{r+t}{-r}) = f(-1) = ?$
 $f(u) = u^r - r$ $a = -1$ $(-1)^r - r(-1) = 1 + r = 3$
 $f(-1) = 3$

$y = \sqrt{u^r + au^r + bu - 1}$ $[a, b]$ D_f $u \rightarrow \dots$
 $y = \sqrt{(u+r)^r} \rightarrow u^r = -1 \rightarrow u = -r$
 $(u-r)^r = u^r - 4u^r + 12u - 1 \rightarrow y = \sqrt{(u-r)^r} \rightarrow y = u - r$
 $a = -4$ $b = 12$
 $-1 \leq u \leq 12 \rightarrow -1 \leq u - r \leq 1 \rightarrow R_f = [-1, 1]$

$y = [a] + [a + \frac{1}{r}]$ $[\frac{1}{r}, r]$

$\frac{1}{r} \leq u < 1 \rightarrow 0 + 1 = 1$
 $1 \leq u < \frac{r}{r} \rightarrow 1 + 1 = 2$
 $\frac{r}{r} \leq u < r \rightarrow 1 + r = r$

$[1, r]$

