

الف) $f(x) = 2 \sin^2(x + \frac{\pi}{2}) - 1$: $\rightarrow -1 \rightarrow 1 \rightarrow 2-1=1 \rightarrow R_f = [-1, 1]$

ب) $g(x) = 3 \cos^3 x - 4$: $\rightarrow 3-4 = -1 \rightarrow -3-4 = -7 \rightarrow R_g = [-7, -1]$

$y = \frac{x}{|x|} + [x] \rightarrow R_f = [-3, 3]$ $\rightarrow x \geq 0 \rightarrow [0, 3] : \frac{x}{x} + [x] = 1 + [x]$ $\rightarrow x < 0 \rightarrow [-3, 0) : \frac{x}{-x} + [x] = -1 + [x]$

فواصل : $\rightarrow$

$[0, 1) : 1$
 $[1, 2) : 2$
 $[2, 3) : 3$
 $\{3\} : 4$
 $[-1, 0) : -2$
 $[-2, -1) : -3$
 $[-3, -2) : -4$

$y = \frac{x+1}{x+3}$: فواصل : $y = \frac{x+1}{x+3}$ $y = \frac{x+1}{x+3}$ $y = \frac{x+1}{x+3}$

عبارت یکتا : $\rightarrow$ $y = \frac{x+1}{x+3}$ $y = \frac{x+1}{x+3}$ $y = \frac{x+1}{x+3}$

عبارت افقی : $\rightarrow$ $y = \frac{1}{3}$

$|x-2| + |y-1| = 3$: روش مادی : $\rightarrow$ $\begin{cases} y = 2-2 = 4 \\ y = 2-4 = -2 \\ y = 2+2 = 4 \\ y = -2 = -2 \end{cases}$

روش : $\begin{cases} x \geq 2 : x-2 + |y-1| = 3 \rightarrow |y-1| = 5-x \rightarrow \begin{cases} y = 2-x \\ y = x-4 \end{cases} \\ x < 2 : -x+2 + |y-1| = 3 \rightarrow |y-1| = x+1 \rightarrow \begin{cases} y = 2+x \\ y = -x \end{cases} \end{cases}$

$\rightarrow x = 5 \rightarrow \begin{cases} y = 2-5 = -3 \\ y = 5-4 = 1 \end{cases} / x = -1 \rightarrow \begin{cases} y = 2-(-1) = 3 \\ y = -(-1) = 1 \end{cases}$

الف) $f(x) = x + \frac{1}{x} + 2\sqrt{x+\frac{1}{x}}$: $\rightarrow x + \frac{1}{x} \geq 0 \rightarrow \frac{x^2+1}{x} \geq 0 \rightarrow \frac{1}{x} \geq 0 \rightarrow x > 0$

$y = a + \frac{1}{a} \rightarrow R_y = [2, +\infty)$ $\rightarrow 2\sqrt{x+\frac{1}{x}} = 2(\sqrt{x+\frac{1}{x}}) = [2\sqrt{x+\frac{1}{x}}]$

$R_f = [2, +\infty) + [2\sqrt{x+\frac{1}{x}}, +\infty) \Rightarrow R_f = [2(1+\sqrt{2}), +\infty)$

ب) $y = (3 - \sin x)(1 + \sin x)$: $\rightarrow y = -\sin^2 x + 2\sin x + 3$ $\rightarrow -1+2+3 = 4$ $\rightarrow -1-2+3 = 0$ $\rightarrow R_y = [0, 4]$

$\frac{b}{2a} = \frac{-2}{-2} = 1$

$$f(n) = \frac{1}{n} + 1 \rightarrow D_f = \{1, 2, 3, \dots, 9\}$$

$$\hookrightarrow R_f = \left\{ \frac{1}{1} + 1, \frac{1}{2} + 1, \frac{1}{3} + 1, \dots, \frac{1}{9} + 1 \right\} \rightarrow (1 \times 9) + \frac{(1+2+\dots+9)}{9} = 9 + 5 = 14 = \underline{14}$$

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$$f(n) = \sqrt{an^2 + bn + c} \rightarrow D_f = [r, +\infty)$$

$$\Rightarrow f(r) = \sqrt{rb + c} = 1 \rightarrow \begin{cases} rb + c = 1 \\ r^2b + c = 0 \end{cases} \rightarrow \begin{cases} b = 1 \\ c = -r \end{cases}$$

$$f(n) = \sqrt{bn + c} \Rightarrow \frac{r}{-r + c} = \boxed{b}$$

$$g(n) = \sqrt{bn^2 + an + c} = \sqrt{n^2 + c} \rightarrow R_g = \sqrt{[E_0 + \infty)} = [r, +\infty)$$

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$$f(n) = \frac{n^2 - 4n}{n + 2} = \frac{n(n-4)}{n+2} \rightarrow D_f = \mathbb{R} - \{a, b\}$$

$$\Rightarrow n(n-4) = 0 \rightarrow n^2 - 4n = 0 \rightarrow (n-4)(n+2) = 0 \rightarrow n = 4, -2 \Rightarrow \boxed{a, b = 4, -2}$$

$$\hookrightarrow \begin{cases} x_1 = \frac{-b}{2a} = \frac{-(-4)}{2(1)} = 2 \\ x_2 = (1) \pm \sqrt{(-4)^2 - 4(1)(-2)} = -1 \end{cases} \rightarrow R_f = [-1, +\infty) - \{4\} \Rightarrow \boxed{c = -1}$$

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$$y = \sqrt[3]{n^3 + an^2 + bn - 1} \xrightarrow{\text{نقطة صفر}} \sqrt[3]{(n-1)^3} = n-1 \Rightarrow y = \sqrt[3]{(n-1)^3} = \underline{n-1}$$

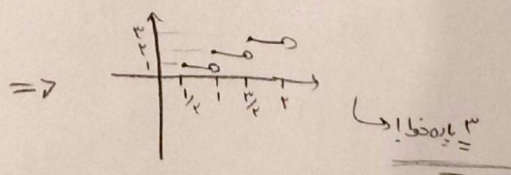
$$\hookrightarrow (n-1)^3 = n^3 - 3n^2 + 3n - 1 \rightarrow \begin{cases} a = -3 \\ b = 3 \end{cases}$$

$$\rightarrow D_f = [-1, 1] \rightarrow \begin{cases} y = -1 - 1 = -2 \\ y = 1 - 1 = 0 \end{cases} \Rightarrow R_f = \underline{[-2, 0]}$$

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$$y = [n] + \left[n + \frac{1}{n}\right] \rightarrow D_f = \left[\frac{1}{n}, 2\right)$$

$$\begin{aligned} n = \left[\frac{1}{n}\right] &\rightarrow \left[\frac{1}{n}\right] + \left[\frac{1}{n} + 1\right] = 1 \\ n = \left[1, \frac{1}{n}\right] &\rightarrow \left[1, \frac{1}{n}\right] + \left[1, \frac{1}{n} + 1\right] = 2 \\ n = \left[\frac{1}{n}, 1\right] &\rightarrow \left[\frac{1}{n}, 1\right] + \left[\frac{1}{n}, 1 + 1\right] = 2 \end{aligned}$$



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