

الف) $f(x) = 2 \sin^2(x + \frac{\pi}{2}) - 1$: $\rightarrow -1 \rightarrow 1 \rightarrow 2-1=1 \rightarrow R_f = [1, 1]$

ب) $g(x) = 3 \cos^3 x - 4$: $\rightarrow 3-4=-1 \rightarrow -3-4=-7 \rightarrow R_g = [-7, -1]$

$y = \frac{x}{|x|} + [x] \rightarrow D_f = [-3, 3]$: $\rightarrow x \geq 0 \rightarrow [0, 3] : \frac{x}{|x|} + [x] = 1 + [x]$
 $\rightarrow x < 0 \rightarrow [-3, 0) : \frac{x}{|x|} + [x] = -1 + [x]$

نمودار :

صفر تعریف شده است!

$y = \frac{x+1}{x+3}$: نمودار :

عبارت یکنوازی : $y = \frac{x+1}{x+3} = 1 - \frac{2}{x+3}$

عبارت افقی : $y = 1$

$|x-2| + |y-1| = 3$: روش مادی :

روش : $\begin{cases} x \geq 2 : x-2 + |y-1| = 3 \rightarrow |y-1| = 5-x \rightarrow \begin{cases} y = 2-x \\ y = x-4 \end{cases} \\ x < 2 : -x+2 + |y-1| = 3 \rightarrow |y-1| = x+1 \rightarrow \begin{cases} y = 2+x \\ y = -x \end{cases} \end{cases}$

$\rightarrow x=5 \rightarrow \begin{cases} y=2-5=-3 \\ y=5-4=1 \end{cases} / x=-1 \rightarrow \begin{cases} y=2-1=1 \\ y=-(-1)=1 \end{cases}$

ا) $f(x) = x + \frac{1}{x} + 2\sqrt{x+\frac{1}{x}}$: $\rightarrow x + \frac{1}{x} \geq 0 \rightarrow \frac{x^2+1}{x} \geq 0 \rightarrow \frac{1}{x} \geq 0 \rightarrow x > 0$

$y = a + \frac{1}{a} \rightarrow R_y = [2, +\infty)$: $\rightarrow 2\sqrt{x+\frac{1}{x}} = 2(\sqrt{x} + \frac{1}{\sqrt{x}}) = 2\sqrt{x} + \frac{2}{\sqrt{x}}$

$R_f = [2, +\infty) + [2\sqrt{x} + \frac{2}{\sqrt{x}}, +\infty) \Rightarrow R_f = [2(1+\sqrt{2}), +\infty)$

ب) $y = (3 - \sin x)(1 + \sin x)$: $\rightarrow y = -\sin^2 x + 2\sin x + 3$

$\rightarrow R_y = [-3, 4]$

$$f(n) = \frac{1}{n} + 1 \rightarrow D_f = \{1, 2, 3, \dots, 9\}$$

$$\hookrightarrow R_f = \left\{ \frac{1}{1} + 1, \frac{1}{2} + 1, \frac{1}{3} + 1, \dots, \frac{1}{9} + 1 \right\} \rightarrow (1 \times 9) + \frac{(1+2+\dots+9)}{9} = 9 + 5 = 14$$

$$f(n) = \sqrt{an^2 + bn + c} \rightarrow D_f = [r, +\infty)$$

$$\Rightarrow f(r) = \sqrt{ar^2 + br + c} = 1 \rightarrow \begin{cases} ar^2 + br + c = 1 \\ ar^2 + br + c = 0 \end{cases} \rightarrow \begin{cases} b = 1 \\ c = -1 \end{cases}$$

$$f(n) = \sqrt{bn + c} \Rightarrow \frac{1}{-b} + \frac{1}{b} = 0 \rightarrow b = 1$$

$$g(n) = \sqrt{bn^2 + an + c} = \sqrt{n^2 + c} \rightarrow R_g = \sqrt{[c, +\infty)} = [r, +\infty)$$

$$f(n) = \frac{n^2 - 4n}{n + 2} = \frac{n(n-4)}{n+2} \rightarrow D_f = \mathbb{R} - \{a, b\}$$

$$\Rightarrow n(n-4) = 0 \rightarrow n^2 - 4n = 0 \rightarrow (n-4)(n) = 0 \rightarrow n = 4, 0 \Rightarrow a, b = 4, -2$$

$$\hookrightarrow \begin{cases} x_1 = \frac{-b}{2a} = \frac{-(-4)}{2(1)} = 2 \\ x_2 = \frac{(1) \pm \sqrt{1 - (-1)}}{2(1)} = 1 \end{cases} \rightarrow R_f = [-1, +\infty) - \{1\} \Rightarrow c = -1$$

$$\rightarrow f\left(\frac{a+b}{2}\right) = f\left(\frac{4-2}{2}\right) = f(-1) = -1 \left(\frac{1}{-1} - 4\right) = 5$$

$$y = \sqrt[3]{n^3 + an^2 + bn - 1} \rightarrow y = \sqrt[3]{(n-1)^3} = n-1$$

$$\hookrightarrow (n-1)^3 = n^3 - 3n^2 + 3n - 1 \rightarrow \begin{cases} a = -3 \\ b = 3 \end{cases}$$

$$\rightarrow D_f = [-1, 1] \rightarrow \begin{cases} y = -1 - 1 = -2 \\ y = 1 - 1 = 0 \end{cases} \Rightarrow R_f = [1, 1]$$

$$y = [n] + \left[n + \frac{1}{n}\right] \rightarrow D_f = \left[\frac{1}{n}, 2\right)$$

$$\begin{aligned} n = \left[\frac{1}{n}\right] &\rightarrow \left[\frac{1}{n}\right] + \left[\frac{1}{n} + \frac{1}{n}\right] = \frac{1}{n} \\ n = \left[1, \frac{1}{n}\right] &\rightarrow \left[\frac{1}{n}\right] + \left[\frac{1}{n} + \frac{1}{n}\right] = \frac{2}{n} \\ n = \left[\frac{2}{n}, \frac{1}{n}\right] &\rightarrow \left[\frac{2}{n}\right] + \left[\frac{2}{n} + \frac{1}{n}\right] = \frac{3}{n} \end{aligned}$$

