

$$\star f(-a) - a f(a) = 2|a| + 1$$

$$x = +a \rightarrow f(-a) - a f(a) = 2|a| + 1$$

$$x = -a \rightarrow f(a) + a f(-a) = 2|-a| + 1 = 2|a| + 1$$

$$f(-a) - a f(a) = f(a) + a f(-a)$$

$$f(-a) - a f(a) = f(a) + a f(-a)$$

$$(1-a) f(-a) = (1+a) f(a)$$

$$\frac{(1+a)}{(1-a)} f(a) = f(-a)$$

الف) $f(x) = 2|x| + 1$ $\star$ $f(x) - a f(a) = 2|a| + 1$ (1)

$$\star \frac{(1+a)}{(1-a)} f(a) - a f(a) = 2|a| + 1$$

$$f(a) \left(\frac{1+a}{1-a} - a \right) = 2|a| + 1$$

$$f(a) \left(\frac{1+a-a(1-a)}{1-a} \right) = 2|a| + 1$$

$$f(a) = \frac{(2|a| + 1)(1-a)}{a^2 + 1}$$

$$f(a) = \frac{-2a^2 + 2|a| - a + 1}{a^2 + 1}$$

$$f(a) + f(0) = a^2 + 2a + 4 \xrightarrow{a=0} 2 f(0) = 4 \quad f(0) = 2$$

$$f(a) = a^2 - 2a + 2 \quad f(a^2) = a^4 - 2a^2 + 2$$

$$y = \frac{a^2 + a + 2}{a-1} \rightarrow ya - y = a^2 + a + 2 \quad a^2 + (1-y)a + 2+y = 0$$

$$a = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-(1-y) \pm \sqrt{(1-y)^2 - 4(2+y)}}{2}$$

$$y^2 - 2y + 1 - 4y - 4 \geq 0 \quad y^2 - 4y - 3 \geq 0 \quad (y-5)(y+1) \geq 0$$

$$Ry = (-\infty, -1] \cup [5, +\infty)$$

$$b) y = 2a^2 - a + 1 \quad as \rightarrow \frac{a}{\varepsilon} \quad y = 2x \frac{a}{\varepsilon} - a \frac{a}{\varepsilon} + 1 = 1 \quad Ry = [1, +\infty)$$

$$c) y = \sqrt{-a^2 + 4a - 2}$$

$$y' = -a^2 + 4a - 2$$

$$as \rightarrow \frac{-4}{-2} = 2 \quad y = -9 + 14 - 2 = 3$$

$$Ry = \sqrt{[-\infty, 4]} = [0, 2]$$

$$Ry' = (-\infty, 2]$$

$$d) y = \sqrt{a^2 - 4a^2 + 4a + 1}$$

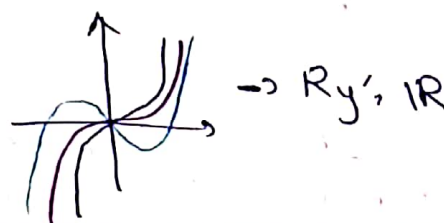
$$y' = a^2 - 4a^2 + 4a + 1$$

$$Ry' = (-\infty, +\infty)$$

$$Ry = \sqrt{(-\infty, +\infty)} = [0, +\infty)$$

$$e) y = \log a^2 - 4a^2 + 4a + 1$$

$$y' = a^2 - 4a^2 + 4a + 1$$



$$\rightarrow Ry' = \mathbb{R}$$

$$Ry = \log \mathbb{R} = \mathbb{R} = (-\infty, +\infty)$$

$$الف) y = \frac{a^2 + 1}{2a - 4} \xrightarrow{a=2} \text{undefined} \quad a = \frac{y y' + 1}{y' - 4} \rightarrow y \neq \frac{1}{4} \quad Ry = \mathbb{R} - \left\{ \frac{1}{4} \right\}$$

$$ب) y = \sqrt{\frac{a^2 + 1}{a - 2}}$$

$$y' = \frac{a^2 + 1}{a - 2}$$

$$a = \frac{y y' + 1}{y' - 2} \rightarrow y' \neq 2 \quad Ry' = \mathbb{R} - \{2\}$$

$$Ry = \sqrt{\mathbb{R} - \{2\}} = [0, +\infty) - \{2\}$$

$$ع.) y = \frac{ax^2 + 1}{\sqrt{ax^2 + 3}} = \frac{ax^2 + 3}{\sqrt{ax^2 + 3}} + \frac{1}{\sqrt{ax^2 + 3}} = \sqrt{ax^2 + 3} + \frac{1}{\sqrt{ax^2 + 3}}$$

$a > 0 \rightarrow \frac{a+1}{a} > 2$
 $a < 0 \rightarrow X$

مقدار $\frac{1}{\sqrt{ax^2 + 3}} = 0 \rightarrow$ مقدار $\rightarrow \min y = \sqrt{3} + \frac{1}{\sqrt{3}} = \frac{4}{\sqrt{3}}$ $Ry = [\frac{4}{\sqrt{3}}, +\infty) = (\frac{4}{\sqrt{3}}, +\infty)$

$>) y = \frac{ax^2 + 1}{ax^2 + 3} \rightarrow y = \frac{ax^2 + 3}{ax^2 + 3} + \frac{1}{ax^2 + 3} - 1$

$Ry = [\frac{1}{3}, +\infty)$

$\frac{a+1}{a} > a > 0 \rightarrow \checkmark$
 $\frac{a+1}{a} < a < 0 \rightarrow X$

$\frac{a^2}{\sqrt{ax^2 + 3}} = 0 \rightarrow \min y = 0 + \frac{1}{\sqrt{3}} - \epsilon = \frac{1}{\sqrt{3}} - \epsilon = \frac{1}{\sqrt{3}}$

$y = \frac{3ax^2 + 2}{x^2 - 1} \rightarrow ① y = \frac{3ax^2 + 2}{x^2 - 1} \rightarrow (y - y_0)(x^2 - 1) = 0 \rightarrow x^2 = \frac{0 - 1(y - y_0)(1 + y_0)}{y(y - y_0)}$

$y \neq 3 \quad ①$

$-1(y - y_0)(1 + y_0) > 0 \rightarrow 4y + 4y^2 - 1 - 12y > 0 \rightarrow 4y^2 - 8y - 1 > 0$

$y^2 + y - 12 > 0$

① $\cap$ ② $\rightarrow (-\infty, -\frac{1}{4}) \cup (3, +\infty)$

$(y + \frac{1}{4})(y - 3) > 0$

$\frac{-\frac{1}{4}}{+} \frac{3}{-} = +$

$(-\infty, -\frac{1}{4}) \cup (3, +\infty) \quad ②$

$\frac{2}{x^2 - 1} \rightarrow x^2 = +\infty \rightarrow \frac{x + \infty}{+ \infty} = 2$

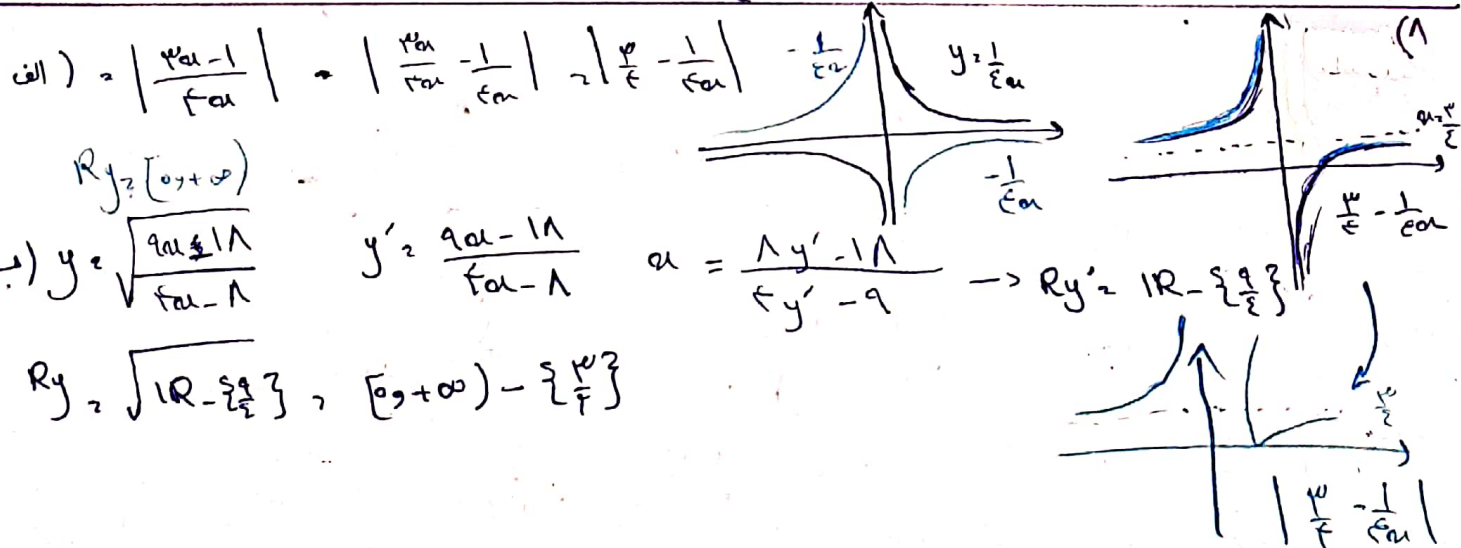
$x > 0 \rightarrow \frac{x}{-1} = -1$

$Ry = (-\infty, -\frac{1}{4}) \cup (3, +\infty)$

$y = \frac{2 \sin x + 1}{\sin x - 3} \rightarrow \sin x = 1 \rightarrow \frac{4}{-2} = -2$

$\sin x = -1 \rightarrow \frac{-2 + 2}{-2} = \frac{0}{-2} = 0$

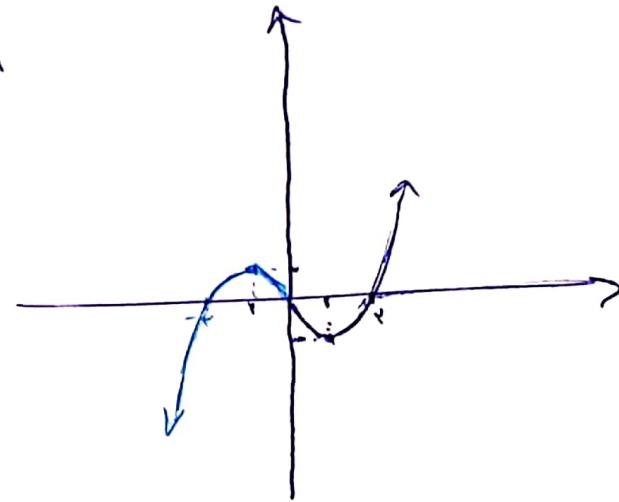
$Ry = [-2, -\frac{1}{3}]$



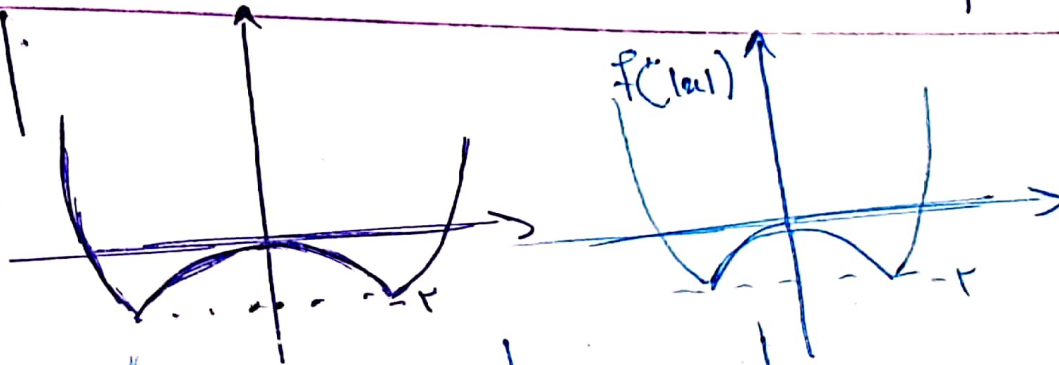
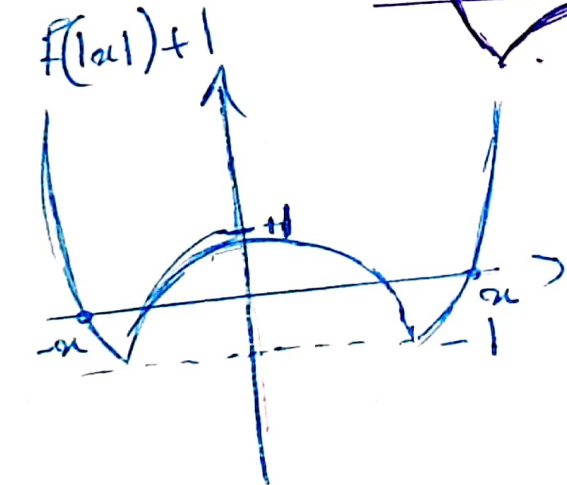
$$y = a|x| - r; \quad x \in \mathbb{R}; \quad a > 0; \quad r > 0; \quad y \in \mathbb{R}$$

$$y = -a|x| - r; \quad x \in \mathbb{R}; \quad a > 0; \quad r > 0; \quad y \in \mathbb{R}$$

$$R_y = \mathbb{R}; \quad (-\infty, +\infty)$$



$$y = |f(|x|) + 1|$$



$$g(x) = |f(|x|) + 1|$$

