

$$\star f(-a) - af(a) = 2|a| + 1$$

1A, VA

برای هر عدد a داریم $f(a) - af(a) = 2|a| + 1$

$$x = +a \rightarrow f(-a) - af(a) = 2|a| + 1$$

$$x = -a \rightarrow f(a) + af(-a) = 2|-a| + 1 = 2|a| + 1$$

$$f(-a) - af(a) = f(a) + af(-a)$$

$$f(-a) - af(a) = f(a) + af(-a)$$

$$(1-a)f(-a) = (1+a)f(a)$$

$$\frac{(1-a)}{(1-a)} f(a) = f(-a)$$

$$\star \frac{(1+a)}{(1-a)} f(a) - af(a) = 2|a| + 1 \quad (1)$$

$$f(a) \left(\frac{1+a}{1-a} - a \right) = 2|a| + 1 \quad (2)$$

$$f(a) \left(\frac{1+a-a(1-a)}{1-a} \right) = 2|a| + 1$$

$$f(a) = \frac{(2|a| + 1)(1-a)}{a^2 + 1}$$

$$f(a) = \frac{-2a^2 + 2|a| - a + 1}{a^2 + 1}$$

$$f(a) + f(0) = a^2 + 2a + 4 \xrightarrow{a=0} 2f(0) = 4 \quad f(0) = 2$$

$$f(a) = a^2 - 2a + 2 \quad f(a^2) = a^4 - 2a^2 + 2$$

$$y = \frac{a^2 + a + 2}{a-1} \rightarrow ya - y = a^2 + a + 2 \quad a^2 + (1-y)a + 2-y = 0$$

$$a = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-(1-y) \pm \sqrt{(1-y)^2 - 4(2-y)}}{2}$$

$$y^2 - 2y + 1 - 4y - 4 \geq 0 \quad y^2 - 4y - 3 \geq 0 \quad (y-2)(y+1) \geq 0$$

$$R_y = (-\infty, -1] \cup [3, +\infty)$$

$$a) y = 2a^2 - 2a + 1 \quad a \in \mathbb{R} \quad y' = 4a - 2 \quad a = \frac{y+2}{4} \quad y = 2\left(\frac{y+2}{4}\right)^2 - 2\left(\frac{y+2}{4}\right) + 1 = \frac{y^2 + 4y + 4}{8} - \frac{y+2}{2} + 1 = \frac{y^2 - 4y + 4}{8} = \frac{(y-2)^2}{8}$$

$$y = \sqrt{-a^2 + 4a - 2}$$

$$y' = -a^2 + 4a - 2$$

$$a = \frac{-4 \pm \sqrt{16 - 4(-2)}}{-2} = \frac{-4 \pm \sqrt{24}}{-2} = -2 \pm \sqrt{6}$$

$$R_y = \sqrt{(-\infty, 2]} = [0, 2]$$

$$R_{y'} = (-\infty, 2]$$

$$c) y = \sqrt{a^2 - 2a^2 + 2a + 1}$$

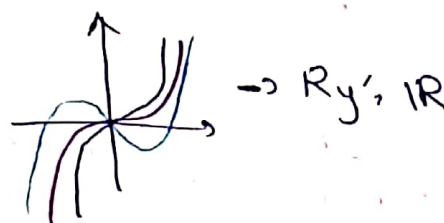
$$y' = a^2 - 2a^2 + 2a + 1$$

$$R_{y'} = (-\infty, +\infty)$$

$$R_y = \sqrt{(-\infty, +\infty)} = [0, +\infty)$$

$$d) y = \log a^2 - 4a^2 + 2a + 2$$

$$y' = 2a - 4a^2 + 2a + 2$$



$$R_{y'} = \mathbb{R}$$

$$R_y = \log \mathbb{R} = \mathbb{R} = (-\infty, +\infty)$$

$$الف) y = \frac{2a+1}{2a-4} \xrightarrow{a=2} \text{undefined} \quad a = \frac{y+1}{2y-4} \rightarrow y \neq \frac{1}{2} \quad R_y = \mathbb{R} - \{\frac{1}{2}\}$$

$$ب) y = \sqrt{\frac{2a+1}{a-2}} \quad y' = \frac{2a+1}{a-2} \xrightarrow{a=2} \text{undefined} \quad a = \frac{y'+1}{y'-2} \rightarrow y' \neq 2 \quad R_{y'} = \mathbb{R} - \{2\}$$

$$R_y = \sqrt{\mathbb{R} - \{2\}} = [0, +\infty) - \{2\}$$

$$ع.) y = \frac{ax^2 + 1}{\sqrt{ax^2 + 3}} = \frac{ax^2 + 3}{\sqrt{ax^2 + 3}} + \frac{1}{\sqrt{ax^2 + 3}} = \sqrt{ax^2 + 3} + \frac{1}{\sqrt{ax^2 + 3}}$$

$a > 0 \rightarrow \frac{a+1}{a} > 2$
 $a < 0 \rightarrow X$

مقدار $= 0 \rightarrow$ مقدار $\rightarrow \min y = \sqrt{3} + \frac{1}{\sqrt{3}} = \frac{4}{\sqrt{3}}$ $Ry = [\frac{4}{\sqrt{3}}, +\infty) = (\frac{4\sqrt{3}}{3}, +\infty)$

$y = \frac{ax^2 + 1}{ax^2 + 3} \rightarrow y = \frac{ax^2 + 3}{ax^2 + 3} + \frac{1}{ax^2 + 3} - 1$

$Ry = [\frac{1}{3}, +\infty)$

$a > 0 \rightarrow \frac{a+1}{a} > 2$
 $a < 0 \rightarrow X$

$\min y = 0 + \frac{1}{3} - 1 = -\frac{2}{3}$
 $\frac{1}{3} - 1 = -\frac{2}{3}$

$y = \frac{3ax^2 + 2}{x^2 - 1} \rightarrow ① \quad yx^2 - y = 3ax^2 + 2 \quad (y-y) \frac{ax^2 + 1}{a} = 0 \quad ax = \pm \frac{\sqrt{0 - 1(y-y)(1+y)}}{y(y-y)} \quad (y \neq 3) ①$

$-1(y-y)(1+y) > 0 \quad 4y + 1y^2 - 1 - 12y > 0 \quad 1y^2 + 1y - 1 > 0$

(2)

① ② $\rightarrow (-\infty, -\frac{1}{2}) \cup (3, +\infty)$

$(y+1)(y-3) > 0$
 $\frac{-1}{2} \quad 3$
 $(-\infty, -\frac{1}{2}) \cup (3, +\infty) ②$

$\frac{3}{x^2} \rightarrow ax^2 = +\infty \rightarrow \frac{x + \infty}{+\infty} = 3$

$\rightarrow \frac{1}{-1} = -1 \rightarrow$ منبر سورا $Ry = (-\infty, -\frac{1}{2}) \cup (3, +\infty)$

$y = \frac{2 \sin x + 1}{\sin x - 3} \rightarrow \sin x = 1 \rightarrow \frac{4}{-2} = -2$

$\sin x = -1 \rightarrow \frac{-2+1}{-2} = \frac{-1}{-2} = \frac{1}{2}$

$Ry = [-\frac{1}{2}, \frac{1}{2}]$ (2) (3)

الف) $| \frac{3ax-1}{fax} | = | \frac{3ax-1}{fax} - \frac{1}{fax} | = | \frac{3}{f} - \frac{1}{fax} | = \frac{1}{\epsilon a}$

$Ry = [0, +\infty)$

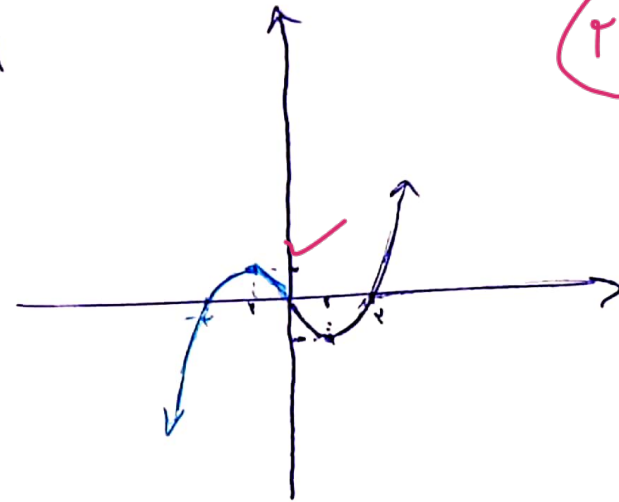
ب) $y = \sqrt{\frac{9ax-1}{fax-1}}$ $y' = \frac{9ax-1}{fax-1}$ $a = \frac{1y'-1}{fy'-9} \rightarrow Ry = \{ \frac{1}{f} \}$

$Ry = \sqrt{1R - \frac{9}{f^2}} \quad [0, +\infty) - \frac{1}{f}$

1- ب) $y = \sqrt{\frac{9(n-1)}{f(n-1)}} = \sqrt{\frac{9}{f}} = \frac{3}{\sqrt{f}} \rightarrow Ry = \{ \frac{3}{\sqrt{f}} \}$

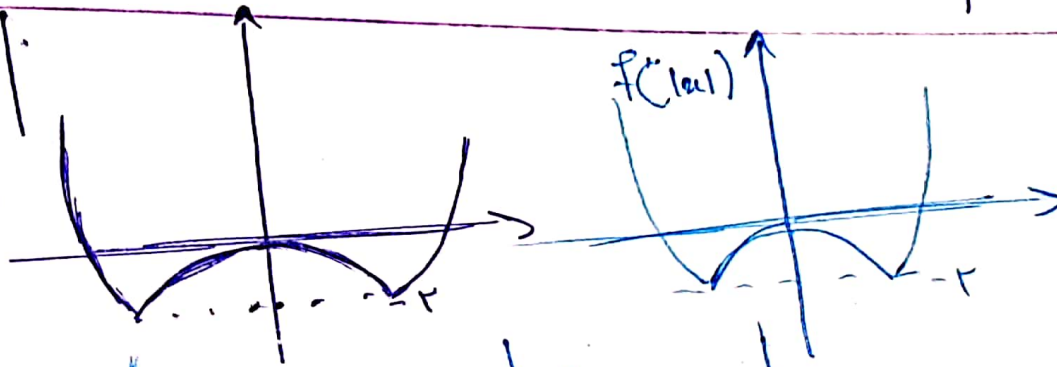
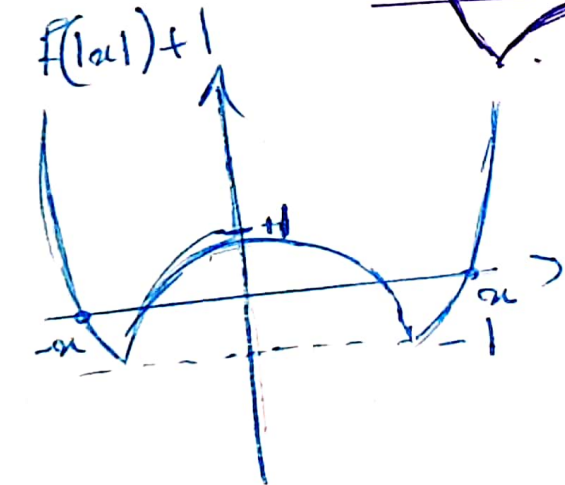
$$y = a|x| - r; \quad a > \frac{r}{r} \geq 1 \quad y > -1$$

$$\begin{aligned} & \xrightarrow{a < 1} -a|x| - r; \quad a < \frac{r}{r} < 1 \quad y < -1 \\ & R_y = \mathbb{R} \quad (-\infty, +\infty) \end{aligned}$$

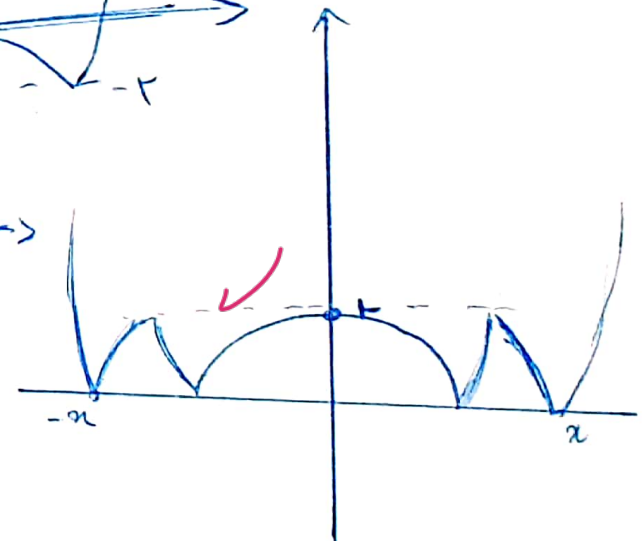


(2) (9)

$$y = |f(|x|) + 1|$$



$$g(x) = |f(|x|) + 1| \rightarrow$$



(2) (1)

۴- یک سهم داریم پس از محققات رأس استفاده می‌کنیم:

الف $S \Big|_{-\frac{17}{\lambda}}^{\frac{5}{\lambda}} \rightarrow R_y = [-\frac{17}{\lambda}, +\infty)$