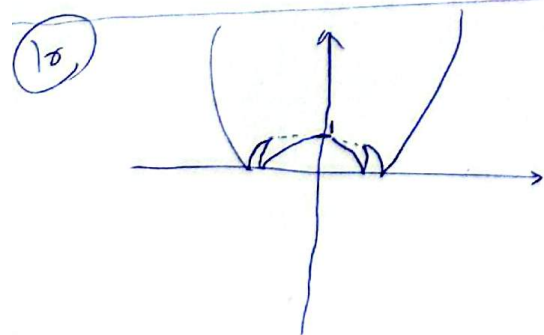


$(4) f(x) = y, \frac{1}{x^2-1} \rightarrow y, \frac{1}{x^2-1} > 1, \frac{1}{x^2-1}$
 $y < 1 \rightarrow y < 1 \rightarrow y < 1 \rightarrow y < 1$
 $x^2 < \frac{y+1}{y-1} \rightarrow \frac{y+1}{y-1} > 0$
 $R = (-\infty, 1) \cup [1, +\infty)$

$(5) z = \sin x \rightarrow y, \frac{1}{z-1} \geq 1, \frac{1}{z-1} \rightarrow z \in [-1, 1] \rightarrow y-1 \in [-1, -1]$
 $z > 1 \Rightarrow y < -1 \rightarrow z > 1 \Rightarrow y < -1 \Rightarrow R_y = [-1, -\frac{1}{1}]$

$(6) f(x) \rightarrow R_y = [-1, +\infty)$
 $y = \sqrt{\frac{9(x-1)}{x-1}} \geq \sqrt{\frac{9}{1}} \geq R_y = \left\{ \frac{3}{1} \right\}$

$(7) y = x|x| - 2x$
 $x > 0 \rightarrow x^2 - 2x$
 $x = 0 \rightarrow 0$
 $x < 0 \rightarrow -x^2 - 2x$
 $R_y = \mathbb{R}$



$$\begin{aligned}
 1) \quad & f(-x) - xf(x) = 2|x| + 1 \rightarrow f(-x) = 2|x| + 1 + xf(x) \\
 & f(x) + xf(-x) = 2|x| + 1 \rightarrow f(x) + x(2|x| + 1 + xf(x)) = 2|x| + 1 \\
 & \Rightarrow (1+x^2)f(x) = (2|x|+1)/(1-x) \rightarrow f(x) = \frac{(2|x|+1)(1-x)}{1+x^2}
 \end{aligned}$$

$$\begin{aligned}
 2) \quad & x_{10} \rightarrow 2f(0) = 4 \rightarrow f(0) = 2 \\
 & f(x) + 2 = x^2 - 2x + 4 \rightarrow f(x) = x^2 - 2x + 2 \xrightarrow{x \rightarrow x^2} f(x^2) = x^4 - 2x^2 + 2
 \end{aligned}$$

$$\begin{aligned}
 3) \quad & y = \frac{x^2+x+2}{x-1} \rightarrow x^2-y, x^2+x+2 \rightarrow x^2+(1-y)x+2+y \\
 & x = \frac{-(1-y) \pm \sqrt{(1-y)^2 - 4(2+y)}}{2} \Delta \geq 0 \rightarrow (1-y)^2 - 4(2+y) \geq 0 \rightarrow y^2 + 1 - 2y - 8 - 4y \geq 0 \\
 & \quad \quad \quad y^2 - 4y - 7 \geq 0 \\
 & \quad \quad \quad (y+1)(y-7) \geq 0 \\
 & \quad \quad \quad y \leq -1 \vee y \geq 7 \\
 & R_y = (-\infty, -1] \cup [7, +\infty)
 \end{aligned}$$

$$\begin{aligned}
 4) \quad & \text{a) } y = 2x^2 - \omega x + 1 \begin{cases} x_2 = -\frac{b}{2a} = \frac{\omega}{4} \\ y_2 = -\frac{1\omega}{4} \end{cases} \xrightarrow{a > 0} R_y = [-\frac{1\omega}{4}, +\infty) \\
 & \text{b) } y = \sqrt{-x^2+4x-\omega} \rightarrow \begin{cases} x_2 = -\frac{b}{2a} = \frac{2}{1} = 2 \\ y_2 = \varepsilon \end{cases} \xrightarrow{a < 0} R_y = \sqrt{(-\infty, \varepsilon]} = [0, 2] \\
 & \text{c) } y = \sqrt{x^2-2x^2+\varepsilon x+1} \rightarrow R_y = [0, +\infty) \\
 & \text{d) } y = \log(x^2-4x^2+vx+\varepsilon) \rightarrow R_y = (-\infty, +\infty)
 \end{aligned}$$

$$\begin{aligned}
 5) \quad & \text{a) } y = \frac{2x+1}{x^2-4} \rightarrow R_y = R - \left\{ \frac{a}{c} \right\} \rightarrow R_y = R - \left\{ \frac{1}{4} \right\} \\
 & \text{b) } y = \sqrt{\frac{\varepsilon x+1}{x-2}} \rightarrow R_y = \sqrt{R - \{\varepsilon\}} = [0, +\infty) - \{2\}
 \end{aligned}$$

$$6) \quad y = \frac{x^2+\varepsilon}{\sqrt{x^2+2}} \rightarrow \frac{x^2+2+1}{\sqrt{x^2+2}} = \sqrt{x^2+2} + \frac{1}{\sqrt{x^2+2}} \rightarrow R_y = [2, +\infty)$$

$$\begin{aligned}
 & y = \frac{1}{x^2+\varepsilon} \rightarrow \frac{1}{x^2+\varepsilon} + \frac{1}{x^2+\varepsilon} - \varepsilon \rightarrow a > 0 \\
 & \frac{1}{x^2+\varepsilon} - \varepsilon \geq -2 \rightarrow R_y = [-2, +\infty)
 \end{aligned}$$