

1) $f(-x) - xf(x) = 2|x| + 1 \rightarrow f(-x) = 2|x| + 1 + xf(x)$
 $f(x) + xf(-x) = 2|x| + 1 \rightarrow f(x) + x(2|x| + 1 + xf(x)) = 2|x| + 1$
 $\Rightarrow (1+x^2)f(x) = (2|x|+1)/(1-x) \rightarrow f(x) = \frac{(2|x|+1)(1-x)}{1+x^2}$ ✓

2) $x_{10} \rightarrow 2f(0) = 4 \rightarrow f(0) = 2$
 $f(x) + 2 = x^2 - 2x + 4 \rightarrow f(x) = x^2 - 2x + 2 \xrightarrow{x \rightarrow x^2} f(x^2) = x^4 - 2x^2 + 2$ ✓

3) $y = \frac{x^2+x+2}{x-1} \rightarrow x^2+y, x^2+x+2 \rightarrow x^2+(1-y)x+2+y$
 $x = \frac{-(1-y) \pm \sqrt{(1-y)^2 - 4(2+y)}}{2} \Delta \geq 0 \rightarrow (1-y)^2 - 4(2+y) \geq 0 \rightarrow y^2 + 1 - 2y - 8 - 4y \geq 0$
 $y^2 - 4y - 7 \geq 0$
 $(y+1)(y-7) \geq 0$
 $R_y = (-\infty, -1] \cup [7, +\infty)$ ✓

4) a) $y = 2x^2 - \omega x + 1 \begin{cases} x_2 = -\frac{b}{2a} = \frac{\omega}{4} \\ y_2 = -\frac{1\omega}{4} \end{cases} \xrightarrow{a > 0} R_y = [-\frac{1\omega}{4}, +\infty)$
 b) $y = \sqrt{-x^2+4x-\omega} \rightarrow \begin{cases} x_2 = -\frac{b}{2a} = \frac{2}{1} = 2 \\ y_2 = \varepsilon \end{cases} \xrightarrow{a < 0} R_y = \sqrt{(-\infty, \varepsilon]} = [0, 2]$
 c) $y = \sqrt{x^2-2x^2+\varepsilon x+1} \rightarrow R_y = [0, +\infty)$
 d) $y = \log(x^2-4x^2+vx+\varepsilon) \rightarrow R_y = (-\infty, +\infty)$ ✓

5) a) $y = \frac{2x+1}{x-2} \rightarrow R_y = R - \left\{\frac{a}{c}\right\} \rightarrow R_y = R - \left\{\frac{2}{-1}\right\} = R - \{-2\}$
 b) $y = \sqrt{\frac{\varepsilon x+1}{x-2}} \rightarrow R_y = \sqrt{R - \{\varepsilon\}} = [0, +\infty) - \{2\}$ ✓

6) a) $y = \frac{x^2+\varepsilon}{\sqrt{x^2+2}} \rightarrow \frac{x^2+2+\varepsilon}{\sqrt{x^2+2}} = \sqrt{x^2+2} + \frac{1}{\sqrt{x^2+2}} \rightarrow R_y = [2, +\infty)$ ✓

b) $y = \frac{1}{x^2+\varepsilon} \rightarrow \frac{1}{x^2+\varepsilon} + \varepsilon \rightarrow a > 0$
 $\frac{1}{x^2+\varepsilon} - \varepsilon \geq -1 \rightarrow R_y = [-1, +\infty)$ ✓

روش اولی: $y n^r - y = 3n^r + 4 \rightarrow (y-3)n^r = y+4$ -4

$$n^r = \frac{y+4}{y-3} \geq 0 \rightarrow \frac{-4}{+} \frac{3}{-} \Rightarrow R_y = (-\infty, -4] \cup (3, +\infty)$$

روش دوم: $\square = n^r \rightarrow \begin{cases} n^r = 0 \rightarrow y = -4 \\ n^r \sim \infty \rightarrow y = 3 \end{cases} \xrightarrow[\text{صفه شصت}]{\text{مخرج بیستون}} R_y = (-\infty, -4] \cup (3, +\infty)$

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$$\rightarrow y = n^r + 4 + \frac{1}{n^r + 4} - 4 \xrightarrow{a > 0} \min a = 4 \rightarrow y_{\min} = \frac{1}{4}$$

$$\Rightarrow R_y = \left[\frac{1}{4}, +\infty \right)$$