

۱)

$$f(-n) - n f(n) \leq r|n| + 1, f(n) \leq ?$$

$$\begin{aligned} n \leq n &\Rightarrow f(-n) - n f(n) \leq r|n| + 1 \xrightarrow{x(-n)} -n f(-n) + n f(n) \leq r|n| - n \\ n \leq -n &\Rightarrow f(n) + n f(-n) \leq r|n| + 1 \\ |n| \leq -n &\Rightarrow \frac{f(n) + n f(-n)}{(n+1)f(n)} \leq \frac{r(-n)|n| - n + 1}{1+n^2} \Rightarrow f(n) \leq \frac{r|n|(1-n) - n + 1}{1+n^2} \end{aligned}$$

$$f(n) + f(0) \leq n^2 + n + 1, f(n^2) \leq ?$$

$$n \leq 0 \Rightarrow f(0) + f(0) \leq 1 \Rightarrow f(0) \leq \frac{1}{2}$$

$$n \leq n^2 \Rightarrow f(n^2) + 1 \leq n^2 + n^2 + 1 \Rightarrow f(n^2) \leq n^2 + n^2 + 1$$

۲)

$$y \leq \frac{n^2 + n + 1}{n-1} \rightarrow y n - y \leq n^2 + n + 1$$

$$n^2 + (1-y)n + r + y \leq 0 \Rightarrow n \leq \frac{(1-y) \pm \sqrt{(1-y)^2 - 4(r+y)}}{2}$$

$$(1-y)^2 - 4(r+y) \geq 0$$

$$y^2 + 1 - 2y \Rightarrow y^2 - 2y - 1 \geq 0$$

$$\frac{-1}{+} \quad \frac{1}{-}$$

$$R_y \leq (-\infty, -1] \cup [1, +\infty)$$

۳)

$$a) y \leq r n^2 - a n + 1 \rightarrow \text{ext} \left\{ \begin{aligned} \frac{1}{4a} \leq n \leq \frac{a}{r} \\ y \leq r \left(\frac{a}{r} \right)^2 - a \left(\frac{a}{r} \right) + 1 = \frac{a^2}{r} - \frac{a^2}{r} + \frac{1}{r} = \frac{1}{r} \end{aligned} \right. \Rightarrow R_y \leq \left[\frac{1}{r}, +\infty \right)$$

۴)

$$b) y \leq \sqrt{-n^2 + n - a} \xrightarrow{a \leq 0} \text{ext} \left\{ \begin{aligned} n \leq \frac{-y}{-1} \leq r \\ y \leq -a + n - a \leq r \end{aligned} \right. \Rightarrow \sqrt{-\infty, r} \Rightarrow R_y \leq [0, r]$$

$$c) y \leq \sqrt{n^2 - r n^2 + n + 1} \rightarrow \sqrt{r} \Rightarrow R_y \leq [0, +\infty)$$

$$d) \log \frac{(n^2 - r n^2 + n + 1)}{1r} \Rightarrow R_y \leq \mathbb{R}$$

با توجه به شکل نمودار برداریم در بازه‌های $(-\infty, 0)$ و $(0, +\infty)$ است

$$a) y \leq \frac{r n + 1}{n - r} \xrightarrow{\text{ad} \neq bc} R_y \leq \mathbb{R} - \left\{ \frac{r}{r} \right\}$$

$$b) y \leq \frac{r n + 1}{n - r} \xrightarrow{\text{ad} \neq cb} \sqrt{\mathbb{R} - \{r\}} \Rightarrow R_y \leq [0, +\infty) - \{r\}$$

۵)

$$c) y \leq \frac{n^2 + r}{\sqrt{n^2 + r}} \xrightarrow{\text{ad} \neq 0} \sqrt{r} + \frac{1}{\sqrt{r}} \leq \frac{r}{r} = \frac{1}{r} \Rightarrow R_y \leq \left[\frac{1}{r}, +\infty \right)$$

$$d) y \leq n^2 + \frac{1}{n^2 + r} + r - r = \frac{n^2 + r}{n^2 + r} + \frac{1}{n^2 + r} - r \xrightarrow{\text{ad} \neq 0} \frac{1}{r} \leq \frac{1}{r} \Rightarrow R_y \leq \left[\frac{1}{r}, +\infty \right)$$

۶)

$$y \leq \frac{n^2 + r}{n^2 - 1}$$

$$\frac{0}{0} \Rightarrow \frac{n^2 + r}{n^2 - 1} \xrightarrow{\text{ad} \neq 0} \frac{r}{-1} \leq -r \Rightarrow R_y \leq (-\infty, -r] \cup [r, +\infty)$$

چون مضرب به نای $n = \pm 1$ صفری نشود پس صفری در تابع این دو است

$$\frac{r}{0} \Rightarrow y \leq \frac{n^2 + r}{n^2 - 1} \xrightarrow{\text{ad} \neq 0} \frac{r}{-1} \leq -r \Rightarrow R_y \leq (-\infty, -r] \cup [r, +\infty)$$

برای جوابی

$$f(x) = \frac{r \sin x + r}{\sin x - r}$$

$$\frac{aQ+b}{cQ+d}$$

$$\sin x = 1$$

$$\sin x = -1$$

$$\frac{-r+r}{-1-r} \leq \frac{r}{-r} \leq \frac{1}{r}$$

$$Ry \leq [-r, -\frac{1}{r}]$$

(داخل و خارج)

$$\frac{r+r}{1-r} \leq \frac{r}{-r} \leq -r$$

$$-1 \leq \sin x \leq 1$$

$$-r \leq \sin x - r \leq -2$$

مخرج می تواند منفی باشد

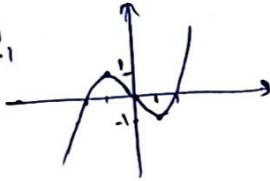
$$y \leq \left| \frac{ax-1}{x} \right| \xrightarrow{ad \neq bc} |R - \frac{1}{x}| \Rightarrow Ry \leq [0, +\infty) - \left\{ \frac{1}{x} \right\}$$

$$R \leq \mathbb{R} - \left\{ \frac{a}{c} \right\}$$

$$y = \sqrt{\frac{9x-11}{-x+2x}} = \sqrt{\frac{9(x-2)}{x(x-2)}} = \sqrt{\frac{9}{x}} = \frac{3}{\sqrt{x}} \Rightarrow Ry \leq \left\{ \frac{3}{\sqrt{x}} \right\}$$

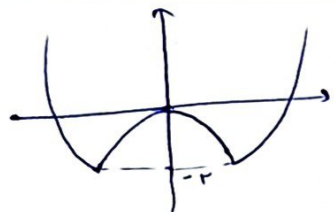
$$y \leq x|x| - rx \xrightarrow{x \rightarrow r} x(x-r)$$

$$y \leq x|x| - rx \begin{cases} x^2 - rx & x \geq 0 \rightarrow \text{ext} \begin{cases} 1 \\ -1 \end{cases} \\ -x^2 - rx & x < 0 \rightarrow \text{ext} \begin{cases} 1 \\ -1 \end{cases} \end{cases}$$



$$Ry \leq \mathbb{R}$$

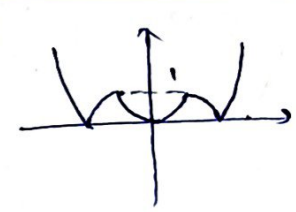
$$f(x) \leq$$



$$f(|x|) + 1$$



$$|f(|x|) + 1|$$



7

8

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