

①

$$-ax f(-a) - a f(a) = 2|a| + 1 \quad f(a) = ?$$

$$f(a) + a f(-a) = 2|a| + 1$$

$$a < 0 \rightarrow f(a) = \frac{2a^2 - a - 2a + 1}{a^2 + 1} = \frac{2a^2 - 3a + 1}{a^2 + 1}$$

$$-a f(-a) + a^2 f(a) = -2a|a| - a$$

$$a > 0 \rightarrow f(a) = \frac{2a + 1 - 2a^2 - a}{a^2 + 1} = \frac{-2a^2 + a + 1}{a^2 + 1}$$

$$f(a) + a f(-a) = 2|a| + 1$$

$$a^2 f(a) + f(a) = -2a|a| - a + 2|a| + 1$$

$$f(a)(a^2 + 1) = -2|a| - a + 2|a| + 1 \Rightarrow f(a) = \frac{-2|a| - a + 2|a| + 1}{a^2 + 1}$$

②

$$f(a^2) = 9 \quad f(a) + f(0) = a^2 - 3a + 4$$

$$f(a^2) = a^4 - 3a^2 + 4$$

$$a = 0 \rightarrow f(0) + f(0) = 4 \Rightarrow f(0) = 2 \rightarrow f(a) + 2 = a^2 - 3a + 4 \rightarrow f(a) = a^2 - 3a + 2$$

$$y = \frac{a^2 + a + 1}{a - 1} \quad ya - y = a^2 + a + 1 \rightarrow a^2 + (1-y)a + (1+y) = 0 \rightarrow a = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow \Delta = b^2 - 4ac$$

$$(1-y)^2 - 4(1+y) \geq 0 \rightarrow y^2 - 2y + 1 - 4 - 4y \geq 0 \rightarrow y^2 - 6y - 3 \geq 0$$

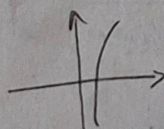
$$\Rightarrow Ry = (-\infty, -1] \cup [7, +\infty)$$

③

$$y = 2a^2 - 2a + 1 \quad \begin{cases} a_S = \frac{-b}{2a} = \frac{1}{2} \\ y_S = 2\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right) + 1 = \frac{1}{2} - 1 + 1 = \frac{1}{2} \end{cases} \xrightarrow{\text{min}} a > 0 \rightarrow Ry = \left[-\frac{1}{2}, +\infty\right)$$

$$y = \sqrt{-a^2 + 4a - 2}$$

$$\begin{cases} a_S = \frac{-b}{2a} = \frac{-4}{-2} = +2 \\ y_S = 2 \end{cases} \Rightarrow \sqrt{(-\infty, 4]} \Rightarrow Ry = [2, 2]$$



$$y = \sqrt{a^2 - 2a^2 + 4a + 1} \rightarrow Ry = [0, +\infty)$$

$$y = \log(a^2 - 4a^2 + 4a + 1) \rightarrow Ry = (-\infty, +\infty)$$

④

$$y = \frac{a^2 + 1}{a^2 - 4} \rightarrow Ry = R - \left\{ \frac{1}{4} \right\}$$

$$y = \frac{a^2 + 1}{\sqrt{a^2 + 1}} = \sqrt{a^2 + 1} + \frac{1}{\sqrt{a^2 + 1}} \Rightarrow Ry = \left[\frac{2}{\sqrt{2}}, +\infty \right)$$

$$y = \sqrt{\frac{a^2 + 1}{a - 2}} \rightarrow Ry = [0, +\infty) - \left\{ \frac{1}{2} \right\}$$

$$y = \frac{a^2 + 1}{a^2 + 1} - 1 \Rightarrow Ry = \left[\frac{1}{2}, +\infty \right)$$

⑤

$$y = \frac{2a^2 + 1}{a^2 - 1} \xrightarrow{\text{ob.}} a^2 = \frac{-y - 1}{y - 2} = \frac{y + 1}{y - 2} \rightarrow a = \pm \sqrt{\frac{y + 1}{y - 2}} \Rightarrow Ry = (-\infty, -1] \cup [1, +\infty)$$

$$a^2 = 0 \rightarrow y = \frac{1}{-1} = -1$$

$$a^2 = +\infty \rightarrow y = \frac{2}{1} = 2$$

$$\hookrightarrow Ry = (-\infty, -1] \cup [2, +\infty)$$

حرف متواتر می شود (یعنی $a = \pm 1$) پس خارج از محدوده است و چون y به ازای $+\infty$ به دست می آید پس باز است.

$$f(x) = \frac{2\sin x + 1}{\sin x - 3}$$

$$\begin{matrix} \xrightarrow{\sin x = +1} \\ \xrightarrow{\sin x = -1} \end{matrix}$$

$$\frac{2+1}{-1-3} = \frac{3}{-4} = -\frac{3}{4}$$

$$\frac{-2+1}{-1-3} = \frac{-1}{-4} = \frac{1}{4}$$

$$\Rightarrow R_f = \left[-\frac{3}{4}, \frac{1}{4}\right]$$

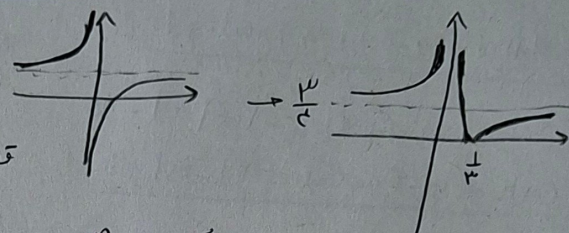
همه خروجی‌ها بین ۰ و ۱ قرار دارند
پس $\sin x$ برابر ۰ نمی‌شود پس
بین ۰ و ۱ قرار دارد

(۷)

$$y = \left| \frac{3a-1}{a} \right|$$

$$\text{دست‌چین} = 0 \rightarrow \text{جانب چپ}$$

$$\text{جانب راست} = \frac{a}{c} = \frac{3}{4}$$



(۸)

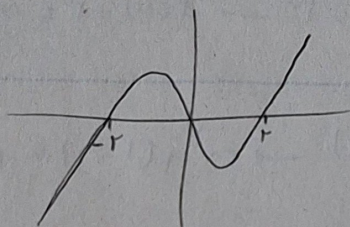
$$\frac{3a-1}{a} = 0 \rightarrow 3a-1=0 \rightarrow a=\frac{1}{3}$$

$$\Rightarrow R_f = [0, +\infty)$$

$$y = \sqrt{\frac{4a+11}{-1+4a}} = \sqrt{\frac{4(a+2.75)}{4(a-0.25)}} \Rightarrow y = \sqrt{\frac{a+2.75}{a-0.25}} = \frac{3}{2} \Rightarrow R_f = \left\{ \frac{3}{2} \right\}$$

$$y = a|a| - 2a$$

$$\begin{array}{l|l} -a^2 - 2a & a^2 - 2a \\ \hline a(-a-2) & a(a-2) \\ \downarrow & \downarrow \\ a = -2 & a = 2 \end{array}$$

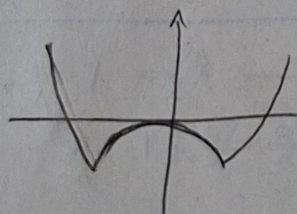


$$R_f = \mathbb{R}$$

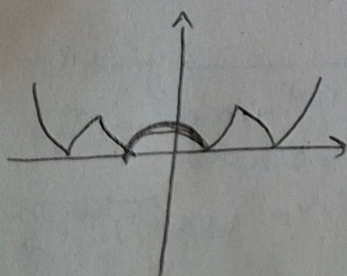
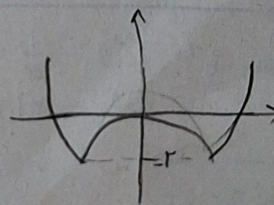
(۹)

$$y = |f(|a|) + 1|$$

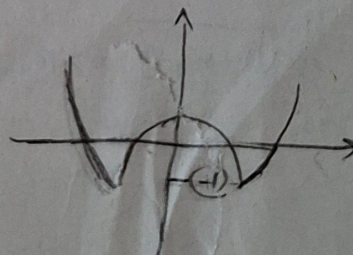
$$\text{①} \rightarrow \text{②} \rightarrow \text{③}$$



همه مقادیر
آنها را در نظر بگیر



نقطه‌های
در صاف یا تیره قرمز



نقطه‌های
افزودن

$$\Rightarrow R_f = [0, +\infty)$$

(۱۰)