

$$f(x) - xf(x) = 2|x| + 1$$

$$f(x) = ?$$

$$\begin{aligned} x \rightarrow f(-x) - xf(x) &= 2|x| + 1 \xrightarrow{+x} xf(-x) - x^2 f(x) = 2|x| + x \\ x \rightarrow f(x) - xf(-x) &= 2|-x| + 1 \xrightarrow{+x} f(x) - xf(-x) = 2|-x| + 1 \end{aligned} \Rightarrow f(x) + x^2 f(x) = (1-x)|x| + 1 - x|x|$$

$$f(x)(1+x^2) = 2|-x| + 1 - x|x| - x \rightarrow f(x) = \frac{1-|x| + 1 - x|x|}{1+x^2}$$

$$f(x) + f(0) = x^2 - 3x + 4 \quad (1)$$

$$f(x^2) = ?$$

$$x=0 \rightarrow f(0) + f(0) = 0 - 0 + 4 \rightarrow 2f(0) = 4 \rightarrow f(0) = 2$$

$$x^2 \rightarrow f(x^2) + 3 = (x^2)^2 - 3(x^2) + 4 \rightarrow f(x^2) = x^4 - 3x^2 + 1$$

$$y = \frac{x^2 + x + 1}{x-1} = \frac{(x+1)(x-1)}{x-1} \quad yx - y = x^2, x+1$$

$$yx - x^2 - x = 1 + y \rightarrow x^2 - yx - x + y + 1 = 0 \rightarrow x^2 + x(1-y) + y + 1 = 0$$

$$\rightarrow (1-y)^2 - 4(y+1) = 1 + y^2 - 2y - 4y - 4 = y^2 - 6y - 3 = 0 \quad (y-3)(y+1)$$

$$\begin{matrix} + & - & + \\ \hline & & \end{matrix} \sim \mathbb{R} \cdot (-\infty, -1] \cup [3, \infty)$$

$a) y = \frac{x^2}{x^2 + 1}$ $\left \frac{0}{-\frac{1}{x}} \right $ $R = [-\frac{1}{x}, \infty)$	$b) y = \sqrt{\frac{x+1}{x}}$ $\left \frac{0}{\frac{1}{x}} \right $ $R = [0, 1]$	$c) y = \sqrt{x^2 + 1}$ $\left \frac{0}{\sqrt{x^2 + 1}} \right $ $R = [0, \infty)$	$d) y = \log(x^2 - 1)$ $\log \frac{0}{1}$ $R = \mathbb{R}$
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$a) y = \frac{x+1}{x-1}$ $R = \left\{ \frac{p}{q} \right\}$	$b) y = \sqrt{\frac{x+1}{x}}$ $\sqrt{R = \{1\}}$ $R = [0, \infty) - \{1\}$	$c) y = \frac{x^2 + 1}{x^2 + x}$ $\frac{x^2 + 1}{\sqrt{x^2 + x}}$ $R = \left[\frac{1}{\sqrt{2}}, \infty \right)$	$d) y = \frac{x^2 + 1}{x^2 + x} + \frac{1}{x} - \frac{1}{x}$ $\frac{x^2 + 1}{x^2 + x} + \frac{1}{x} - \frac{1}{x}$ $R = \left[\frac{1}{2}, \infty \right)$
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$$y = \frac{px^2 + \varepsilon}{x^2 - 1}$$

$$x_{\max} = \infty \leadsto y = p$$

$$x_{\min} = 0 \leadsto y = -f$$

$$R = (-\infty, -f] \cup [p, \infty)$$

$$\frac{px^2 + \varepsilon}{x^2 - 1}, y$$

$$x = \pm \sqrt{\frac{y + \varepsilon}{y - p}}$$

$$-\frac{f}{p} < y < p$$

$$R = (-\infty, -\varepsilon] \cup (p, \infty)$$

$$F(x) = \frac{r \sin x + f}{\sin x - p}$$

$$\begin{aligned} \text{Max} \quad \sin x = 1 &\leadsto \frac{f}{-p} = -\frac{f}{p} \\ \text{Min} \quad \sin x = -1 &\leadsto \frac{-p + \varepsilon}{-1 - p} = \frac{p}{-\varepsilon} = -\frac{1}{p} \end{aligned}$$

$$R = [-\frac{f}{p}, -\frac{1}{p}]$$

$$\text{الف) } y = \left| \frac{px - 1}{\varepsilon x} \right|$$

$$R = [0, \infty)$$

$$\text{ب) } y = \sqrt{\frac{qx - 11}{-11 + \varepsilon x}}$$

$$\frac{qx - 11}{-11 + \varepsilon x} \cdot y \leadsto \frac{\varepsilon y + 11}{-11y - 9} \cdot x \leadsto D, IR - \left\{ \frac{1}{11} \right\}$$

$$R = [0, \infty)$$

$$y = x|x| - px$$

$$x^2 - px \quad x > 0$$

$$-x^2 - px \quad x < 0$$

$$R = IR$$

$$y = |F(|x|) + 1|$$

