

$$\begin{aligned}
 f(-n) - n f(n) &= 2|n| + 1 \xrightarrow{x=-n} -n f(-n) + n^2 f(n) = -2n|n| - n \\
 f(n) + n f(-n) &= 2|n| + 1 \\
 \begin{cases} -n f(-n) + n^2 f(n) = -2n|n| - n \\ n f(-n) + f(n) = 2|n| + 1 \end{cases} \\
 &= (n^2 + 1) f(n) = (-n + 1) 2|n| + 1 - n \\
 \rightarrow f(n) &= \frac{2|n|(-n + 1) + (1 - n)}{(n^2 + 1)} = \frac{(1 - n)(2n + 1)}{n^2 + 1}
 \end{aligned}$$

۱

$$\begin{aligned}
 f(n) + f\left(\frac{1}{n}\right) &= n^2 - 3n + 3 \rightarrow f(n) = n^2 - 3n + 3 \\
 \rightarrow n=1: \underbrace{f(1) + f(1)}_{2f(1)} &= \dots + 3 \rightarrow \boxed{f(1) = 3} \hookrightarrow f(n^2) = n^4 - 3n^2 + 3
 \end{aligned}$$

۲

$$\begin{aligned}
 y &= \frac{n^2 + n + 2}{n-1} \Rightarrow y n - y = n^2 + n + 2 \rightarrow n^2 + (1-y)n + (2+y) = 0 \\
 \hookrightarrow n &= \frac{-(1-y) \pm \sqrt{(1-y)^2 - 4(2+y)}}{2} = \frac{y-1 \pm \sqrt{y^2 - 4y - 7}}{2} \\
 (n = \frac{-b \pm \sqrt{\Delta}}{2a}) & \Rightarrow y^2 - 4y - 7 \geq 0 \Rightarrow \frac{-1 \pm \sqrt{33}}{2} \\
 \Rightarrow R_f &= (-\infty, -1] \cup [7, +\infty)
 \end{aligned}$$

۳

$$\begin{aligned}
 \text{الف) } y &= \frac{n^2 - 2n + 1}{n-1} : y_2 = \frac{\Delta}{4a} = \frac{1 - 4 \cdot 0}{1} = -\frac{11}{1} \\
 \hookrightarrow R_f &= [-\frac{11}{1}, +\infty) \\
 \text{ب) } y &= \sqrt{-n^2 + 4n - 5} : (-n+2)(n-1) \rightarrow \frac{1}{-1} \frac{\infty}{1} \\
 \hookrightarrow y &= \sqrt{[1, 5]} \rightarrow R_f = [1, \sqrt{5}] \\
 \text{ج) } y &= \sqrt{n^2 - 3n^2 + 4n + 1} : \mathbb{R} \rightarrow y = \sqrt{1R} \\
 R_f &= [0, +\infty) \\
 \text{د) } y &= \log(n^2 - 2n^2 + 4n + 1) : \mathbb{R} \\
 \Rightarrow y &= \log(1R \rightarrow (0, +\infty)) \rightarrow R_f = [1, +\infty)
 \end{aligned}$$

۴

$$\begin{aligned}
 \text{الف) } y &= \frac{n^2 + 1}{n-2} : R_f = \mathbb{R} - \left\{ \frac{5}{4} \right\} \\
 \text{ب) } y &= \sqrt{\frac{4n+1}{n-2}} : y = \sqrt{[2, +\infty) - \{ \frac{1}{4} \}} \\
 \Rightarrow R_f &= [2, +\infty) - \{ \frac{1}{4} \} \\
 \text{ج) } y &= \frac{n^2 + 4}{\sqrt{n^2 + 3}} : \sqrt{n^2 + 3} = a \\
 \hookrightarrow y &= \frac{a^2 + 1}{a} = a + \frac{1}{a} \geq 2 \rightarrow R_f = [2, +\infty) \\
 \text{د) } y &= n^2 + \frac{1}{n^2 + 4} : \frac{+\infty}{+\infty} + \frac{1}{+\infty} = +\infty \\
 \Rightarrow R_f &= [\frac{1}{4}, +\infty)
 \end{aligned}$$

۵

$$y = \frac{3x^2 + \varepsilon}{x^2 - 1} : \text{1) حال اول: } yx^2 - y = 3x^2 + \varepsilon \rightarrow (x-y)x^2 + (\varepsilon+y) = 0$$

$$\rightarrow x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{\pm \sqrt{4(\varepsilon+y)(y+\varepsilon)}}{2(\varepsilon+y)} \rightarrow \text{...}$$

$$\Rightarrow R_f = (-\infty, -\varepsilon] \cup (\varepsilon, +\infty)$$

6

$$2) \text{ حال دوم: } \frac{3x^2 + \varepsilon}{x^2 - 1} = r$$

$$\rightarrow R_f = (-\infty, -\varepsilon] \cup (\varepsilon, +\infty)$$

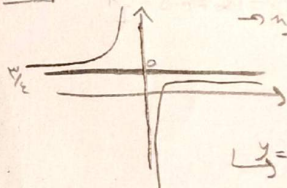
$$f(x) = \frac{x \sin x + \varepsilon}{\sin x - r} : \frac{+1}{-1} \rightarrow \frac{r+\varepsilon}{1-r} = \frac{r}{-r} = -1$$

$$\frac{-r+\varepsilon}{-1-r} = \frac{r}{-\varepsilon} = -\frac{1}{r} \rightarrow R_f = [-r, -\frac{1}{r}]$$

7

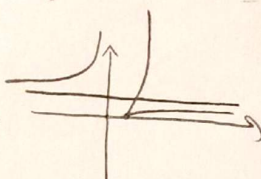
$$\text{الف) } y = \left| \frac{x^2 - 1}{x} \right| :$$

$$y \geq 0$$



$$\rightarrow x=0 \rightarrow y = \frac{1}{2}$$

$$\rightarrow y=0$$



$$\Rightarrow R_f = [0, +\infty)$$

$$\text{ب) } y = \sqrt{\frac{a(x-1)}{x(x-1)}} :$$

$$y \geq 0$$

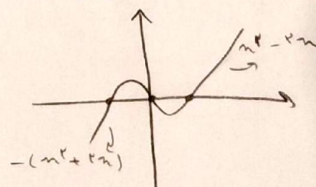
$$\rightarrow y = \sqrt{\frac{a(x-1)}{x(x-1)}} = \sqrt{\frac{a}{x}} \Rightarrow R_f = \left\{ \frac{x}{a} \right\}$$

8

$$y = x|x| - x^2 \rightarrow x \geq 0 : x^2 - x^2 = 0$$

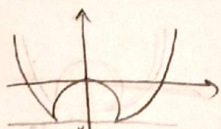
$$\rightarrow x < 0 : -x^2 - x^2 = -2x^2$$

$$\Rightarrow R_f = \mathbb{R}$$

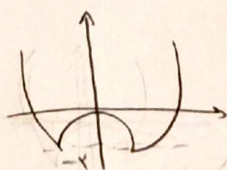


9

$$f(x) : \rightarrow y = |f(x)| + 1 = ?$$

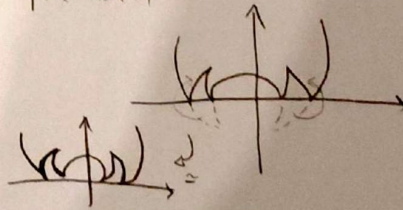
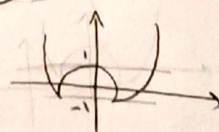


$$\rightarrow 1) y = f(|x|) : \Rightarrow$$



$$\rightarrow 2) y = |f(x)| + 1 :$$

$$\rightarrow 3) y = f(x) + 1 : \Rightarrow$$



10