

۱۸/۲۵ قسمت برد رویه بار دین به عنوان ۱

$$\begin{aligned} f(-n) - n f(n) &= 2|n| + 1 \xrightarrow{x=-n} -n f(-n) + n^2 f(n) = -2n|n| - n \\ f(n) + n f(-n) &= 2|n| + 1 \\ \begin{cases} -n f(-n) + n^2 f(n) = -2n|n| - n \\ n f(-n) + f(n) = 2|n| + 1 \end{cases} \\ &= (n^2 + 1) f(n) = (-n + 1) 2|n| + 1 - n \\ \rightarrow f(n) &= \frac{2|n|(-n + 1) + (1 - n)}{(n^2 + 1)} = \frac{(1 - n)(2|n| + 1)}{n^2 + 1} \checkmark \end{aligned}$$

(۱,۷۵)

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$$\begin{aligned} f(n) + f\left(\frac{1}{n}\right) &= n^2 - 3n + 2 \rightarrow f(n) = n^2 - 3n + 2 \\ \rightarrow n=1: f(1) + f(1) &= \dots + 2 \rightarrow f(1) = 1 \checkmark \rightarrow f(n^2) = n^4 - 3n^2 + 2 \checkmark \end{aligned}$$

(۲)

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$$\begin{aligned} y &= \frac{n^2 + n + 2}{n - 1} \Rightarrow y n - y = n^2 + n + 2 \rightarrow n^2 + (1 - y)n + (2 + y) = 0 \\ \rightarrow n &= \frac{-(1 - y) \pm \sqrt{(1 - y)^2 - 4(2 + y)}}{2} = \frac{y - 1 \pm \sqrt{y^2 - 4y - 7}}{2} \\ (n = \frac{-b \pm \sqrt{\Delta}}{2a}) \quad \rightarrow y^2 - 4y - 7 \geq 0 \rightarrow y \leq -1 \vee y \geq 7 \\ \Rightarrow R_f &= (-\infty, -1] \cup [7, +\infty) \end{aligned}$$

(۳)

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$$\begin{aligned} \text{الف) } y &= \frac{n^2 - 5n + 1}{n - 1} : y_5 = \frac{-\Delta}{2a} = \frac{1 - 20}{2} = -\frac{19}{2} \\ \rightarrow R_f &= [-\frac{19}{2}, +\infty) \\ \text{ب) } y &= \sqrt{-n^2 + 4n - 5} : (-n + 5)(n - 1) \rightarrow \frac{1}{-1} \frac{\infty}{-1} \\ \rightarrow y &= \sqrt{[1, 5]} \rightarrow R_f = [1, 5] \\ \text{ج) } y &= \sqrt{n^2 - 3n^2 + 4n + 1} : \mathbb{R} \rightarrow y = \sqrt{1R} \\ R_f &= [0, +\infty) \\ \text{د) } y &= \log(n^2 - 4n^2 + 5n + 4) : \mathbb{R} \\ \Rightarrow y &= \log(1R) \rightarrow R_f = [0, +\infty) \end{aligned}$$

(۱۲۵)

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$$\begin{aligned} \text{الف) } y &= \frac{n^2 + 1}{n - 2} : R_f = \mathbb{R} - \left\{ \frac{5}{4} \right\} \\ \text{ب) } y &= \sqrt{\frac{4n + 1}{n - 2}} : y = \sqrt{[2, +\infty) - \{ \frac{1}{4} \}} \\ \Rightarrow R_f &= [2, +\infty) - \{ \frac{1}{4} \} \\ \text{ج) } y &= \frac{n^2 + 4}{\sqrt{n^2 + 3}} : \sqrt{n^2 + 3} = a \\ y &= \frac{a^2 + 1}{a} = a + \frac{1}{a} \geq 2 \rightarrow R_f = [2, +\infty) \\ \text{د) } y &= n^2 + \frac{1}{n^2 + 4} : \frac{+\infty}{+\infty} + \frac{1}{+\infty} = +\infty \\ \Rightarrow R_f &= [\frac{1}{4}, +\infty) \end{aligned}$$

(۱,۵)

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با به بیرون یب بشه بعد از ۲ به بعد بیرون به ت
بعضی وقتها مثل اینی a قایم یب شدن توی
نویز و بیرون هست → و به همین خودمون باید حداقل مقدارش پیدا کنیم

$$y = \frac{3x^2 + \varepsilon}{x^2 - 1} : \text{1) اول اول: } yx^2 - y = 3x^2 + \varepsilon \rightarrow (x-y)x^2 + (\varepsilon+y) = 0$$

$$\rightarrow x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{\pm \sqrt{4(y-\varepsilon)(y+\varepsilon)}}{2(x-y)} \rightarrow \dots : \frac{-\varepsilon \pm \sqrt{\varepsilon^2 - (y-\varepsilon)(y+\varepsilon)}}{x-y}$$

$$\Rightarrow R_f = (-\infty, -\varepsilon] \cup (x, \infty)$$

$$2) \text{ دوم اول: } \frac{3x^2 + \varepsilon}{x^2 - 1} = x \rightarrow \frac{3x^2 + \varepsilon}{x^2 - 1} = x \rightarrow 3x^2 + \varepsilon = x(x^2 - 1) \rightarrow 3x^2 + \varepsilon = x^3 - x \rightarrow x^3 - 3x^2 - x - \varepsilon = 0$$

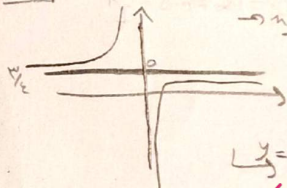
$$R_f = (-\infty, -\varepsilon] \cup (x, \infty)$$

$$f(x) = \frac{x \sin x + \varepsilon}{\sin x - x} : \frac{x \sin x + \varepsilon}{\sin x - x} = y \rightarrow \frac{x \sin x + \varepsilon}{\sin x - x} = y \rightarrow x \sin x + \varepsilon = y(\sin x - x) \rightarrow x \sin x + \varepsilon = y \sin x - xy \rightarrow x \sin x - y \sin x + xy + \varepsilon = 0 \rightarrow (x-y) \sin x + xy + \varepsilon = 0$$

$$\rightarrow R_f = [-x, -\frac{1}{x}]$$

$$\text{الف) } y = \left| \frac{x^2 - 1}{x} \right| :$$

$$y \geq 0$$



$$\rightarrow x=1 \rightarrow y = \frac{1}{2}$$

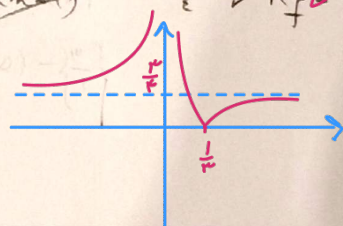
$$L_y = |0| :$$

$$\Rightarrow R_f = [0, \infty)$$

$$\text{ب) } y = \sqrt{\frac{a(x-1)}{x(x-1)}} :$$

$$y \geq 0$$

$$\rightarrow y = \sqrt{\frac{a(x-1)}{x(x-1)}} = \sqrt{\frac{a}{x}} \Rightarrow R_f = \left\{ \frac{x}{a} \right\}$$

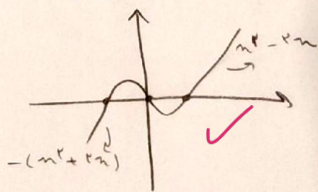


تقریب از اول
طوبه!

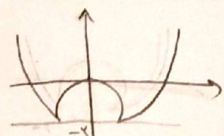
$$y = x|x| - x^2 \rightarrow x \geq 0 : x^2 - x^2 \rightarrow 0$$

$$\rightarrow x < 0 : -x^2 - x^2 \rightarrow -2x^2$$

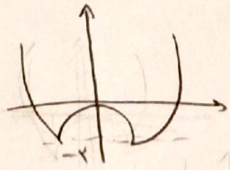
$$\Rightarrow R_f = \mathbb{R}$$



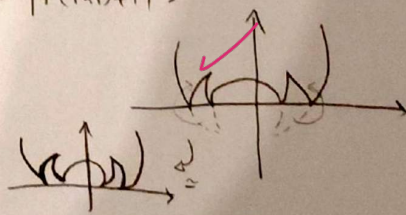
$$f(x) : \rightarrow y = |f(x) + 1| :$$



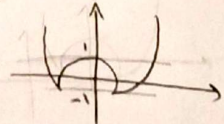
$$\rightarrow 1) y = f(x) + 1 \Rightarrow$$



$$\rightarrow 2) y = |f(x) + 1| :$$



$$\rightarrow 3) y = f(x) + 1 \Rightarrow$$



۴. یک سهم داریم پس از محققات رأس استفاده می‌کنیم:

$$\text{ب) } S \Big|_K^K \Rightarrow R_y = \sqrt{(-\infty, 4]} = [0, 2]$$

$$\text{د) } R_y = \lg \mathbb{R} = (-\infty, +\infty)$$

$$\text{ج) } y = \frac{x^2 + 3 + 1}{\sqrt{x^2 + 3}} = \sqrt{x^2 + 3} + \frac{1}{\sqrt{x^2 + 3}}$$

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$$\xrightarrow{a > 0} \min a = \sqrt{3} \rightarrow y_{\min} = \frac{4\sqrt{3}}{3} \Rightarrow R_y = \left[\frac{4\sqrt{3}}{3}, +\infty \right)$$