

الف) $y''' - y'' = y' - 1 \Rightarrow \begin{cases} y_1''' = y_1'' + y_1' - 1 \\ y_2''' = y_2'' + y_2' - 1 \end{cases} \rightarrow y_1''' = y_2''' \Rightarrow y_1 = y_2$

تابع مستقيم

ب) $n \cos y = y \cos n$ $\frac{n \cos y}{y} = \cos n$ $\leftarrow n = \frac{\pi}{2}$ مثال نقض
 $y = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots$

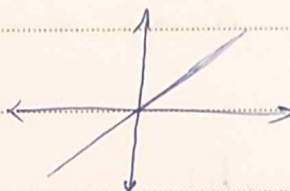
تابع نسبي

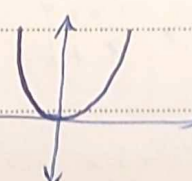
ج) $\frac{n+y}{y} = \sqrt{ny}$, $n, y > 0 \rightarrow n+y - \sqrt{ny} = 0 \rightarrow (n - \sqrt{y})^2 = 0$

$\rightarrow \begin{cases} \sqrt{y_1} = \sqrt{n} \\ \sqrt{y_2} = \sqrt{n} \end{cases} \rightarrow \sqrt{y_1} = \sqrt{y_2} \xrightarrow{\text{تربيع}} |y_1| = |y_2| \xrightarrow{y > 0} y_1 = y_2$

تابع مستقيم

د) $n y'' + y' - y''' - y = 0 \rightarrow n(y'' + y') = y''' + y \rightarrow n = \frac{y(y'' + y')}{y'' + y}$

$y'' = y' \rightarrow n = y$  تابع مستقيم

هـ) الف) $|y| + 2y^2 = y$ $\xrightarrow{y^2 \geq 0, |y| \geq 0} y \geq 0$ $\Rightarrow y + 2y^2 = y \rightarrow y = 2y^2$  تابع مستقيم

ب) $[y] + [n] = 1$ $[y] \leq 1 \rightarrow [n] = 0 \rightarrow n = 0$ مثال نقض
 $\rightarrow y \in \mathbb{Z} \Rightarrow (0, 1), (1, 0), (2, -1), \dots$

تابع نسبي

$$|n-1| \geq r \rightarrow \begin{cases} n-1 \geq r \rightarrow n \geq r+1 \\ n-1 \leq -r \rightarrow n \leq -1 \end{cases}$$

$$f(n) = \begin{cases} r^2 - b & n \geq r, n \leq -1 \\ a + rn & 0 \leq n \leq r \end{cases} \xrightarrow{n=r} r^2 - b = a + rn \rightarrow r^2 - b = a + r \rightarrow \boxed{a + b = r^2}$$

$$f(n) = \sqrt{a - bn - nr} \quad \frac{b}{-} \frac{a}{+} \frac{r}{-} \leftarrow -nr - bn + a \geq 0$$

$$\text{Given } (2) \rightarrow a, b \quad S = a + b = -b \rightarrow a = -r b \xrightarrow{b=1} \boxed{a = -r}$$

$$P = ab = -a \xrightarrow{a \neq 0} \boxed{b = -1}$$

$$\boxed{a + b = 1}$$

$$y = \frac{r}{\sqrt{an^2 + bn + c}} \quad an^2 + bn + c > 0 \quad \frac{r}{+} \frac{0}{+} \frac{-}{-}$$

$$y = bn + c \xrightarrow{n=r, y=0} 0 = rb + c \rightarrow \boxed{c = -rb} \quad \boxed{a \leq 0} \leftarrow \text{or } (2) \checkmark$$

$$\boxed{\text{Given } (2)} \rightarrow b \neq 0$$

$$ra + c - r|b| = 0 - rb \quad r(-b) = +b \rightarrow \boxed{b = -r}$$

$$y = \sqrt{\frac{rn+r}{n+b}} + a \rightarrow \frac{rn+r}{n+b} + a \geq 0 \rightarrow \frac{(r+a)n + r+ab}{n+b} \geq 0$$

$$\frac{r}{-} \frac{0}{+} \frac{+}{+} \quad n = r \text{ or } -r \rightarrow r+b \leq 0 \rightarrow \boxed{b = -r}$$

$$\text{or } (2) \checkmark \rightarrow \frac{r}{+} \frac{0}{+} \frac{+}{+} \rightarrow a + r \geq 0 \rightarrow \boxed{a \geq -r}$$

$$\Rightarrow \frac{1}{n-r} \geq 0 \quad \frac{r}{-} \frac{0}{+} \frac{+}{+}$$

— Arman —

الف) $y = \sqrt{u^2 - u}$ $u^2 - u \geq 0$ $u(u-1) \geq 0$ $u(u+1)(u-1) \geq 0$ ✓

$-\frac{1}{1} + \frac{0}{1} + \frac{1}{1} \Rightarrow D_f = [-1, 0] \cup [1, +\infty)$

—) $y = \frac{1}{\sqrt{|u|-1}}$ $|u|-1 > 0 \rightarrow |u| > 1$ $\begin{cases} u > 1 \\ u < -1 \end{cases}$

$D_f = (-\infty, -1) \cup (1, +\infty)$

الف) $y = \sqrt{\frac{1}{|u| - [u]}}$ $\rightarrow D_f = \mathbb{R} - (\mathbb{Z}^+ \cup \{0\})$ ✓

$|u| = [u] \xrightarrow{u \in \mathbb{Z}} |u| = u \Rightarrow u \geq 0$

—) $\sqrt{u} + \sqrt{y} = 9 \rightarrow \sqrt{y} = 9 - \sqrt{u}$ $[u \geq 0]$

$9 - \sqrt{u} \geq 0 \rightarrow 9 \geq \sqrt{u} \rightarrow 11 \geq |u|$

$D_f = [0, 11]$

$\downarrow u \geq 0$
 $[11 \geq u]$

$[\frac{r_n - 1}{r}] + [\frac{r_n + r - 1}{r}] \geq 0 \rightarrow r[\frac{r_n - 1}{r}] > -r \rightarrow [\frac{r_n - 1}{r}] > -1$ ✓

$\rightarrow 0 < \frac{r_n - 1}{r} \rightarrow \frac{1}{r} < u$ $D_f = [\frac{1}{r}, +\infty)$

$1 - \log(r_n - a) \geq 0 \rightarrow r_n - a \leq 10 \rightarrow n \leq \frac{10 + a}{r}$ $D_f = (b, c]$
 $\downarrow \frac{a}{r}$ $\downarrow \frac{10 + a}{r}$
 $\rightarrow r_n - a > 0 \rightarrow n > \frac{a}{r}$

Arman

$c - b = \frac{10 + a}{r} - \frac{a}{r} = \frac{10}{r}$