

الف) $y = x^2 - \omega \Rightarrow x^2 = y + \omega \Rightarrow x = \pm \sqrt{y + \omega} \rightarrow y + \omega \geq 0 \Rightarrow y \geq -\omega \Rightarrow R_f = [-\omega, +\infty)$ ✓

ب) $y = x^2 + 1 \Rightarrow x^2 = y - 1 \Rightarrow x = \pm \sqrt{y - 1} \rightarrow R_f = [1, +\infty)$ ✓

الف) $y = x^2 - 2x + 3 \Rightarrow y = x^2 - 2x + 1 + 2 = (x - 1)^2 + 2 \Rightarrow (x - 1)^2 = y - 2 \Rightarrow x - 1 = \pm \sqrt{y - 2} \Rightarrow x = \pm \sqrt{y - 2} + 1 \rightarrow y - 2 \geq 0 \Rightarrow y \geq 2 \Rightarrow R_f = [2, +\infty)$ ✓

ب) $y = x^2 - \omega x + 1 \Rightarrow y = (x - \omega/2)^2 - \omega^2/4 + 1 \Rightarrow (x - \omega/2)^2 = y + \omega^2/4 - 1 \Rightarrow x - \omega/2 = \pm \sqrt{y + \omega^2/4 - 1} \Rightarrow x = \pm \sqrt{y + \omega^2/4 - 1} + \omega/2 \rightarrow y + \omega^2/4 - 1 \geq 0 \Rightarrow y \geq -\omega^2/4 + 1 \Rightarrow R_f = [-\omega^2/4 + 1, +\infty)$ ✓
 $\omega/2 = 6/2 = 3$

الف) $y = \frac{x^2 + 3}{x^2 - 2} \Rightarrow yx^2 - 2y = x^2 + 3 \Rightarrow yx^2 - x^2 = 2y + 3 \Rightarrow x^2(y - 1) = 2y + 3 \Rightarrow x^2 = \frac{2y + 3}{y - 1} \Rightarrow x = \pm \sqrt{\frac{2y + 3}{y - 1}} \rightarrow \frac{2y + 3}{y - 1} \geq 0 \rightarrow \frac{-3}{y - 1} \leq 0 \rightarrow y > 1 \Rightarrow R_f = (1, +\infty)$ ✓

ب) $y = \frac{2|x| + 1}{|x| - 2} \Rightarrow yx|x| - 2y = 2|x| + 1 \Rightarrow y|x| - 2|x| = 2y + 1 \Rightarrow |x|(y - 2) = 2y + 1 \Rightarrow |x| = \frac{2y + 1}{y - 2} \Rightarrow x = \pm \frac{2y + 1}{y - 2} \rightarrow \frac{2y + 1}{y - 2} \geq 0 \Rightarrow y < -\frac{1}{2} \text{ or } y > 2 \Rightarrow R_f = (-\infty, -\frac{1}{2}) \cup (2, +\infty)$ ✓

$y = \frac{1}{x^2 - 4x} \Rightarrow yx^2 - 4xy = 1 \Rightarrow yx^2 - 4xy - 1 = 0 \Rightarrow x = \frac{-b \pm \sqrt{\Delta}}{2a} \Rightarrow x = \frac{-(-4y) \pm \sqrt{(-4y)^2 - 4(-y)}}{2y} \Rightarrow x = \frac{4y \pm \sqrt{16y^2 + 4y}}{2y} \rightarrow \Delta \geq 0 \Rightarrow (-4y)^2 - 4(-y) \geq 0 \Rightarrow 16y^2 + 4y \geq 0 \Rightarrow 4y(4y + 1) \geq 0 \Rightarrow y < -\frac{1}{4} \text{ or } y \geq 0 \Rightarrow R_f = (-\infty, -\frac{1}{4}) \cup [0, +\infty)$ ✓

الف) $y = x^2 - 7x + 2 \rightarrow \min_{y=-7} \left\{ \frac{y}{x} = 7 - x \right\} \rightarrow R_f = [-7, +\infty)$ ✓

ب) $y = -x^2 + 6x + 2 \rightarrow \max_{y=6} \left\{ \frac{y}{x} = -x + 6 \right\} \rightarrow R_f = (-\infty, 6]$ ✓

الف) $y = \sqrt{x^2 - 9x + 7} \rightarrow \min \begin{cases} \frac{y}{x} = r = x \\ y = -v \end{cases} \rightarrow R_f = \sqrt{[-v, +\infty)} \Rightarrow R_f = [0, +\infty) \checkmark$

ب) $y = \sqrt{-x^2 + 4x + 10} \rightarrow \max \begin{cases} \frac{-f}{x} = r = x \\ y = 14 \end{cases} \rightarrow R_f = \sqrt{(-\infty, 14]} \Rightarrow R_f = [0, \sqrt{14}] \checkmark \textcircled{2}$

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الف) $y = x^4 + 2x^3 + 2x + 1 \xrightarrow{x^4} R_f = \mathbb{R} \checkmark$

ب) $y = \sqrt{x^4 + 2x^3 + 2x + 1} \xrightarrow{x^4} R_f = \sqrt{\mathbb{R}} \Rightarrow R_f = [0, +\infty) \checkmark \textcircled{2}$

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الف) $y = \frac{2x+1}{x-2} \rightarrow R_f = \mathbb{R} - \left\{\frac{2}{1}\right\} \Rightarrow R_f = \mathbb{R} - \{2\} \checkmark$

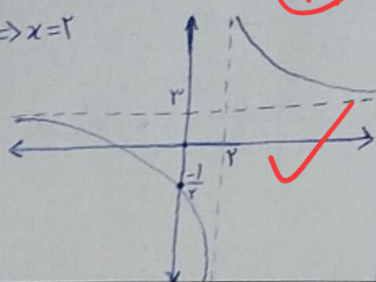
ب) $y = \sqrt{\frac{2x+1}{x-2}} \rightarrow R_f = \sqrt{\mathbb{R} - \left\{\frac{2}{1}\right\}} \Rightarrow R_f = [0, +\infty) - \{2\} \checkmark \textcircled{2}$

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الف) $y = \frac{2x+1}{x-2}$

$\rightarrow x = \{x | x-2=0\} \Rightarrow x=2$

$\rightarrow y = \frac{a}{c} = \frac{2}{1} = 2$



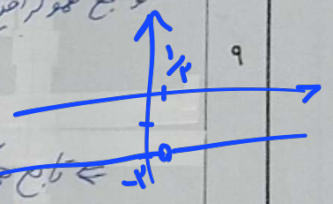
1.5

ب) $y = \frac{2x-2}{1-2x} = \frac{2x-2}{-2x+1} = -2 \quad x \neq \frac{1}{2}$

در توابع همگرافیک $ad \neq bc$ باشد.

$2x-2 \neq -2x+1 \Rightarrow 2 \neq 1$

ب) تابع همگرافیک نمی باشد.



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الف) $y = \cos^2 x + \frac{1}{\cos x} \xrightarrow{\cos x > 0} R_f = [2, +\infty) \checkmark$

ب) $y = \sqrt{\frac{x+1}{x}} \Rightarrow y = \sqrt{x + \frac{1}{x}} \Rightarrow R_f = (-\infty, -2] \cup [2, +\infty) \Rightarrow R_f = [-\infty, -2] \cup [2, +\infty) \checkmark \textcircled{2}$

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