

الف) $y = n^2 - 5 \rightarrow n^2 = y + 5 \rightarrow n = \pm \sqrt{y+5} \rightarrow R_f = [-5, +\infty)$

ب) $y = n^3 + 1 \rightarrow n^3 = y - 1 \rightarrow n = \sqrt[3]{y-1} \rightarrow R_f = \mathbb{R}$

الف) $y = n^2 - 2n + 4$
 $y = (n^2 - 2n + 1) + 3 \rightarrow y - 3 = (n - 1)^2 \rightarrow n - 1 = \pm \sqrt{y - 3} \Rightarrow R_f = [3, +\infty)$

ب) $y = n^2 - 5n + 1$
 $n^2 - 5n + 1 - y = 0 \rightarrow n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{5 \pm \sqrt{25 - 4(1-y)}}{2} \rightarrow R_f = \left[\frac{5}{4}, +\infty\right)$
 $25 - 4(1-y) \geq 0 \rightarrow 4y \geq -21 \rightarrow y \geq -\frac{21}{4}$

الف) $y = \frac{n^2 + 3}{n^2 - 1} \rightarrow n^2 + 3 = y(n^2 - 1) \rightarrow yn^2 - n^2 = y + 3$
 $+ \frac{-3}{-1} \frac{n^2 - 1}{-1} + n^2(y - 1) = y + 3 \rightarrow n^2 = \frac{y + 3}{y - 1} \rightarrow n = \pm \sqrt{\frac{y + 3}{y - 1}}$
 $R_f = (-\infty, -\frac{3}{4}] \cup (1, +\infty)$

ب) $y = \frac{2|n| + 1}{|n| - 1} \rightarrow y|n| - 1 = 2|n| + 1 \rightarrow |n|(y - 2) = 2 + 1$
 $+ \frac{-1}{-1} \frac{|n| - 1}{-1} + |n|(y - 2) = 2 + 1 \rightarrow |n| = \frac{y + 1}{y - 2}$
 $R_f = (-\infty, -\frac{1}{2}] \cup (2, +\infty)$

$y = \frac{1}{n^2 - 4n} \rightarrow yn^2 - 4yn = 1 \rightarrow yn^2 - 4yn - 1 = 0$
 $b^2 - 4ac \geq 0 \Rightarrow \Delta \geq 0$
 $16y^2 + 4y \geq 0 \rightarrow 4y(4y + 1) \geq 0 \rightarrow R_f = (-\infty, -\frac{1}{4}] \cup [0, +\infty)$

الف) $y = n^2 - 4n + 2 \xrightarrow{a > 0} \begin{cases} n_{min} = \frac{-b}{2a} = \frac{4}{2} = 2 \\ y_{min} = -1 \end{cases} R_f = [-1, +\infty)$

ب) $y = -n^2 + 4n + 2 \xrightarrow{a < 0} \begin{cases} n_{max} = \frac{-b}{2a} = \frac{-4}{-2} = 2 \\ y_{max} = 6 \end{cases} R_f = (-\infty, 6]$

الف) $y = \sqrt{x^2 - 4x + 2} \xrightarrow{a > 0} \begin{cases} x_{\min} = \frac{-b}{2a} = 2 \\ y_{\min} = -1 \end{cases} \quad R_f = \sqrt{[-1, +\infty)} = [0, +\infty)$

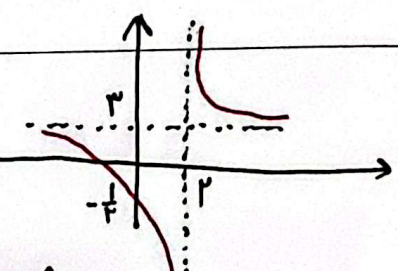
ب) $y = \sqrt{-x^2 + 4x + 10} \xrightarrow{a < 0} \begin{cases} x_{\max} = \frac{-b}{2a} = \frac{-4}{-2} = 2 \\ y_{\max} = \sqrt{14} \end{cases} \quad R_f = \sqrt{(-\infty, 14]} = [0, \sqrt{14}]$

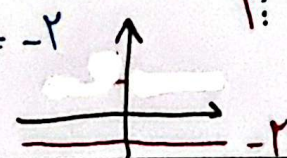
الف) $y = x^5 + 3x^4 + 2x + 1 \rightarrow R_f = \mathbb{R}$

ب) $y = \sqrt{x^5 + 4x^4 + 7x + 1} \rightarrow R_f = [0, +\infty)$

الف) $y = \frac{3x+1}{x-2} \rightarrow R_f = \mathbb{R} - \left\{\frac{a}{c}\right\} = \mathbb{R} - \{3\}$

ب) $y = \sqrt{\frac{3x+1}{x+3}} \rightarrow R_f = \sqrt{\mathbb{R} - \{2\}} \Rightarrow R_f = [0, +\infty) - \{2\}$

الف) $y = \frac{3x+1}{x-2}$
 (2) ریشه‌ی خروج: مجانب قائم $x=2$
 (3) $y = \frac{a}{c}$: مجانب افقی $y=3$


ب) $y = \frac{3x-2}{1-2x} \rightarrow y = \frac{-2(1-2x)}{1-2x} = -2$

 $R_f = \{-2\}$

الف) $y = \cos^2 x + \frac{1}{\cos^2 x} \rightarrow R_f = [2, +\infty)$

ب) $y = \sqrt[3]{\frac{x^2+1}{x}} \rightarrow y = \sqrt[3]{x + \frac{1}{x}}$
 $\frac{x^2+1}{x} = \frac{x}{1} + \frac{1}{x}$
 $R_f = \sqrt[3]{(-\infty, -2] \cup [2, +\infty)}$

$\hookrightarrow R_f = (-\infty, \sqrt[3]{-2}) \cup (\sqrt[3]{2}, +\infty)$