

الف) $y = n^2 - 5 \rightarrow n^2 = y + 5 \rightarrow n = \pm \sqrt{y+5} \rightarrow R_f = [-5, +\infty)$

ب) $y = n^3 + 1 \rightarrow n^3 = y - 1 \rightarrow n = \sqrt[3]{y-1} \rightarrow R_f = \mathbb{R}$

الف) $y = n^2 - 2n + 4$

$y = (n^2 - 2n + 1) + 3 \rightarrow y - 3 = (n-1)^2 \rightarrow n-1 = \pm \sqrt{y-3} \Rightarrow R_f = [3, +\infty)$

ب) $y = n^2 - 5n + 1$

$n^2 - 5n + 1 - y = 0 \rightarrow n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{5 \pm \sqrt{25 - 4(1-y)}}{2} \rightarrow R_f = \left[\frac{5}{4}, +\infty\right)$
 $25 - 4(1-y) \geq 0 \rightarrow 4y \geq -21 \rightarrow y \geq -\frac{21}{4}$

الف) $y = \frac{n^2 + 3}{n^2 - 1} \rightarrow n^2 + 3 = y(n^2 - 1) \rightarrow yn^2 - n^2 = y + 3$

$+ \frac{-3}{-1} - \frac{1}{-1} + \frac{1}{-1} \rightarrow n^2(y-1) = y+3 \rightarrow n^2 = \frac{y+3}{y-1} \rightarrow n = \pm \sqrt{\frac{y+3}{y-1}}$

$R_f = (-\infty, -\frac{3}{2}] \cup (1, +\infty)$

ب) $y = \frac{2|n|+1}{|n|-2} \rightarrow y|n|-2y = 2|n|+1 \rightarrow |n|(y-2) = 2y+1 \rightarrow |n| = \frac{2y+1}{y-2}$
 $R_f = (-\infty, -\frac{1}{2}] \cup (2, +\infty)$

$y = \frac{1}{n^2 - 4n}$

$\rightarrow yn^2 - 4yn = 1 \rightarrow yn^2 - 4yn - 1 = 0$

$\leftarrow b^2 - 4ac \geq 0 \text{ و } \Delta \geq 0$

$16y^2 + 4y \geq 0$

$\rightarrow 4y(4y+1) \geq 0$

$+ \frac{-1}{-4} - \frac{1}{-4} + \frac{1}{-4}$

$R_f = (-\infty, -\frac{1}{4}] \cup (\frac{1}{4}, +\infty)$

الف) $y = n^2 - 4n + 2 \xrightarrow{a > 0}$

$\begin{cases} n_{min} = \frac{-b}{2a} = \frac{4}{2} = 2 \\ y_{min} = -1 \end{cases}$

$R_f = [-1, +\infty)$

ب) $y = -n^2 + 4n + 2 \xrightarrow{a < 0}$

$\begin{cases} n_{max} = \frac{-b}{2a} = \frac{-4}{-2} = 2 \\ y_{max} = 6 \end{cases}$

$R_f = (-\infty, 6]$

الف) $y = \sqrt{x^2 - 4x + 2} \xrightarrow{a > 0} \begin{cases} x_{\min} = \frac{-b}{2a} = 2 \\ y_{\min} = -1 \end{cases} \quad R_f = \sqrt{[-1, +\infty)} = [0, +\infty)$

ب) $y = \sqrt{-x^2 + 4x + 10} \xrightarrow{a < 0} \begin{cases} x_{\max} = \frac{-b}{2a} = \frac{-4}{-2} = 2 \\ y_{\max} = \sqrt{14} \end{cases} \quad R_f = \sqrt{(-\infty, 14]} = [0, \sqrt{14}]$

الف) $y = x^5 + 3x^4 + 2x + 1 \rightarrow R_f = \mathbb{R}$

ب) $y = \sqrt{x^5 + 4x^4 + 7x + 1} \rightarrow R_f = [0, +\infty)$

الف) $y = \frac{3x+1}{x-2} \rightarrow R_f = \mathbb{R} - \left\{\frac{a}{c}\right\} = \mathbb{R} - \{3\}$

ب) $y = \sqrt{\frac{3x+1}{x+3}} \rightarrow R_f = \sqrt{\mathbb{R} - \{3\}} \Rightarrow R_f = [0, +\infty) - \{3\}$

الف) $y = \frac{3x+1}{x-2}$
 (2) ریشه‌ی خروج: $y = \frac{a}{c} (3)$
 محور افقی: $y = \frac{a}{c} (3)$
 محور عمودی: $x = \frac{b}{a} (2)$

ب) $y = \frac{3x-2}{1-2x} \rightarrow y = \frac{-2(1-2x)}{1-2x} = -2$
 $R_f = \{-2\} \quad x \neq \frac{1}{2}$

الف) $y = \cos^2 x + \frac{1}{\cos^2 x} \rightarrow R_f = [2, +\infty)$

ب) $y = \sqrt[3]{\frac{x^2+1}{x}} \rightarrow y = \sqrt[3]{x + \frac{1}{x}}$
 $R_f = \sqrt[3]{(-\infty, -2] \cup [2, +\infty)}$
 $\hookrightarrow R_f = (-\infty, \sqrt[3]{-2}) \cup (\sqrt[3]{2}, +\infty)$