

الف) $y = x^2 - 5 \rightarrow x^2 = y + 5 \rightarrow x = \pm \sqrt{y+5} \rightarrow \text{IR}_f = [-5, +\infty)$

ب) $y = x^2 + 1 \rightarrow x^2 = y - 1 \rightarrow x = \pm \sqrt{y-1} \rightarrow \text{IR}_f = \mathbb{R}$

الف) $y = x^2 - 4x + 4 \rightarrow y = (x^2 - 4x + 4) + 2 \rightarrow y - 2 = (x - 2)^2 \rightarrow x - 2 = \pm \sqrt{y-2} \Rightarrow \text{IR}_f = [2, +\infty)$

ب) $y = x^2 - 2x + 1 \rightarrow x^2 - 2x + 1 - y = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2 \pm \sqrt{4 - 4(1-y)}}{2} = 1 \pm \sqrt{y}$
 $4 - 4(1-y) \geq 0 \rightarrow 4y \geq 0 \rightarrow y \geq 0 \rightarrow y \in \left[-\frac{1}{4}, +\infty\right)$

الف) $y = \frac{x^2 + 3}{x^2 - 1} \rightarrow x^2 + 3 = yx^2 - y \rightarrow yx^2 - x^2 = y + 3 \rightarrow x^2(y-1) = y+3 \rightarrow x^2 = \frac{y+3}{y-1}$
 $\rightarrow x = \pm \sqrt{\frac{y+3}{y-1}}$ $\text{IR}_f = (-\infty, -\frac{2}{3}] \cup (\frac{2}{3}, +\infty)$

ب) $y = \frac{2|x| + 1}{|x| - 4} \rightarrow y|x| - 4y = 2|x| + 1 \rightarrow |x|(y-2) = 4y + 1 \rightarrow |x| = \frac{4y+1}{y-2}$
 $\text{IR}_f = (-\infty, -\frac{1}{4}] \cup (2, +\infty)$

$y = \frac{1}{x^2 - 4x} = yx^2 - 4yx = 1 \rightarrow yx^2 - 4y - 1 = 0 \quad b^2 - 4ac \geq 0 \quad G, A, 2$

$16y^2 + 4y \geq 0 \rightarrow 4y(4y+1) \geq 0$

$\text{IR}_f = (-\infty, -\frac{1}{4}] \cup [0, +\infty)$

الف) $y = x^2 - 4x + 2 \xrightarrow{a > 0} \begin{cases} x_{\min} = -\frac{b}{2a} = \frac{2}{1} = 2 \\ y_{\min} = -4 \end{cases} \quad \text{IR}_f = [-4, +\infty)$

ب) $y = -x^2 + 4x + 2 \xrightarrow{a < 0} \begin{cases} x_{\max} = -\frac{b}{2a} = -\frac{4}{-2} = 2 \\ y_{\max} = 6 \end{cases} \quad \text{IR}_f = (-\infty, 6]$

$$11) y = \sqrt{x^2 - 4x + 1} \xrightarrow{\Delta} \begin{cases} x_{min} = \frac{-b}{2a} = 1 \\ y_{min} = -1 \end{cases} \quad \mathbb{R}_f = \sqrt{[-1, +\infty)} = [0, +\infty)$$

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$$12) y = \sqrt{-x^2 + 2x + 1} \xrightarrow{\Delta} \begin{cases} x_{max} = \frac{-b}{2a} = \frac{-1}{-1} = 1 \\ y_{max} = 1 \end{cases} \quad \mathbb{R}_f = \sqrt{(-1, 1]} \Rightarrow [0, \sqrt{14}]$$

$$13) y = x^4 + 2x^2 + 1 \rightarrow \mathbb{R}_f = \mathbb{R}$$

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$$14) y = \sqrt{x^2 + 4x^2 + 2x + 1} \rightarrow \mathbb{R}_f = [0, +\infty)$$

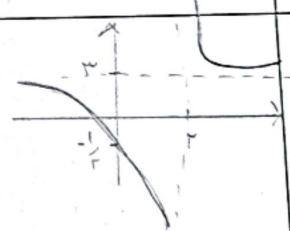
چون زیر رادیکال مثبت

$$الف) y = \frac{3x + 1}{x - 1} \rightarrow \mathbb{R}_f = \mathbb{R} - \left\{ \frac{4}{2} \right\} = \mathbb{R} - \{2\}$$

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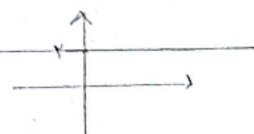
$$ب) y = \sqrt{\frac{x + 1}{x + 2}} \rightarrow \mathbb{R}_f = \sqrt{\mathbb{R} - \{ -2 \}} \Rightarrow \mathbb{R}_f = [0, +\infty) - \{ -2 \}$$

$$الف) y = \frac{3x + 1}{x - 2} \xrightarrow{\text{مشتق}} \begin{cases} \text{مشتق صفر} = \text{مشتق صفر} \\ \text{مشتق منفی} = y = \frac{4}{2} \end{cases}$$



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$$ب) y = \frac{4x - 2}{1 - 2x} \rightarrow y = \frac{-2(1 - 2x)}{1 - 2x} = -2 \quad \mathbb{R}_f = \{-2\}$$



$$الف) y = \cos^2 x + \frac{1}{\cos^2 x} \rightarrow \mathbb{R}_f = [2, +\infty)$$

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$$ب) y = \sqrt[3]{\frac{x^2 + 1}{x}} \rightarrow y = \sqrt[3]{x + \frac{1}{x}} \quad \mathbb{R}_f = \sqrt[3]{(-\infty, -2] \cup [2, +\infty)}$$

$\frac{x^2 + 1}{x} = \frac{x^2}{x} + \frac{1}{x}$